# Math 362
## Practice Exam I

**1.** Find the Cartesian and polar form of the reciprocal to the complex number $z = 3 - 4i$.

***Solution:*** $\dfrac{1}{3-4i} = \dfrac{3+4i}{25} = \dfrac{1}{5}e^{i\theta}$, where $\theta = \arctan\dfrac{4}{3}$.

**2.** Find all cubic roots (in Cartesian and polar form) of $z = 8i$.

***Solution:*** $\dfrac{1}{3-4i} = \dfrac{3+4i}{25} = \dfrac{1}{5}e^{i\theta}$, where $\theta = \arctan\dfrac{4}{3}$.

**3.** What is the polar form for 16 - 2$i$? Find all values of ln (16 -2i) and indicate the principal value Ln (16 -2i).

***Solution:*** 16.1245(cos(– 0.1244) + $i$sin(- 0.1244).

**4.** What is the standard form of the complex number – 13.5(cos(0.58) + $i$sin(0.58))?
***Solution:*** – 11.2922 – 7.3983$i$.

**5.** Find all the roots of $\sqrt[4]{-5}$ .
***Solution:*** 1.0574 + 1.0754$i$, – 1.0574 + 1.0754$i$, 1.0574 – 1.0754$i$, –1.0574 – 1.0754$i$.

**6.** Given $z = 1 - 2i$, what is $e^{z}$?
***Solution:*** $e$(cos(-2) + $i$sin(-2)).

7. Find all values of $(-2i)^{-i}$ and indicate the principal value.
***Solution:*** *exp(-3Pi/2 -2nPi). exp(I ln 2), n ranges over all integers, exp(3Pi/2)exp(I ln 2)*

**8.** For the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

find $A^2$, $A^3$, $A^4$. Do you see a pattern?

***Solution:*** $$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, A^3 = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}, A^4 = \begin{bmatrix} 6 & 5 & 5 \\ 5 & 6 & 5 \\ 5 & 5 & 6 \end{bmatrix}.$$

**9.** Solve the system of algebraic equations by Gauss elimination.

$$\begin{cases} x + 3y - 4z = -2 \\ -2x - y + 2z = 6 \\ 4x - 6y + z = 9 \end{cases}.$$

***Solution:*** $x = 2, y = 0, z = 1.$

**10.** Use Gaussian elimination to obtain the solution of the following system of algebraic equations:

$$\begin{cases} 2x & + & 5y & + & 3z & = & 1 \\ -x & + & 2y & + & z & = & 2. \\ x & + & y & + & z & = & 0 \end{cases}$$

***Solution:*** $x = -1, y = 0, z = 1.$

**11.** Rewrite the following system of algebraic equations

$$\begin{cases} x & - & 2y & + & 2z & = & -6 \\ -2x & + & y & - & 2z & = & 0 \\ 2x & + & 2y & + & z & = & 3 \end{cases}$$

in the matrix form $Ax = b$ and solve it by Gaussian elimination.

***Solution:*** $x = 8, y = -2, z = -9$

**12.** Write the following system in matrix form.

$$x + y + z = 1, x + 2x + 3z = 2, y + z = 3.$$

What are the coefficient matrix $A$ and the augmented matrix $\overline{A}$? What are their ranks? Solve this system.

***Solution:***

$$\overline{A} = \begin{bmatrix} 1 & 1 & 1 & | & 1 \\ 1 & 2 & 3 & | & 2 \\ 0 & 1 & 1 & | & 3 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}, x = -2, y = 5, z = -2. \text{Rank A} = \text{Rank } \overline{A} = 3$$

**13.** For the following linear system determine all values of $a$ for which the resulting linear system has (a) no solutions; (b) a unique solution; (c) infinitely many solutions.

$$x + y + z = 2, 2x + 3x + 2z = 5, 2x + 3y + (a^2 - 1)z = a + 1/.$$

***Solution :*** **(a)** $a = \pm\sqrt{3}$; **(b)** $a \neq \pm\sqrt{3}$; **(c)** There is no value of $a$ such that this system has infinitely many solutions.

**14.** Given the following matrices,

$$A = \begin{bmatrix} 4 & -12 \\ 1 & -3 \\ -3 & 9 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -4 & -1 \end{bmatrix}$$

- **(a)** Multiply matrices $A$ and $B$ to get $AB$.
- **(b)** Does $BA$ exist? Justify your answer.
- **(c)** Are the columns of matrix $A$ linearly independent? Justify your answer.
- **(d)** Find the rank of $A$, $B$, and $AB$.
- **(e)** Do the columns of matrix $A$ span $\mathbf{R}^3$? Justify your answer.

**(f)** Do the columns of matrix $B$ span $\mathbf{R}^2$? Justify your answer.
**(f)** Do the columns of matrix $B$ span $\mathbf{R}^2$? Justify your answer.

***Solution:***

**(a)** $AB = \begin{bmatrix} 56 & 24 \\ 14 & 6 \\ -42 & -18 \end{bmatrix}.$

**(b)** No.

**(c)** No.

**(d)** rank($A$) = 1, rank($B$) = 2, and rank($AB$) = 1.

**(e)** No

**(f)** Yes

**15.** Mark each statement True or False.

**(a)**____In some cases, it is possible for six vectors to span $\mathbf{R}^5$.

**(b)**____If a system of linear equations has two different solutions, then it has infinitely many solutions.

**(c)**____The equation $Ax = b$ is homogeneous if the zero vector is a solution.

**(d)**____If $v_1$ and $v_2$ span a plane in $\mathbf{R}^3$ and if $v_3$ is not in that plane, then $\{v_1, v_2, v_3\}$ is a linearly independent set.

***Solution:***

**(a)** True.

**(b)** True.

**(c)** False.

**(d)** True.