# Review Test 2

## Question 1 A & B

MTH 362

(1)

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & -2 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$R_3 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 & 0 \end{array}\right]$$

$\det A = -2$

$-2w = 0$

$w = 0$

$1z - 1(0) = 0$

$\boxed{z = 0}$

$1y + 0z - 2w = 0$

$\boxed{y = 0}$

$\boxed{x = 1}$

$$D = \begin{vmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -2 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix}$$

(1) B

$$D_1 = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -2 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad -2$$

$$D_2 \quad \begin{vmatrix} 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & -2 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = 0$$

[b/c repeated colom]

Similarly

$$D_3 = D_4 = 0$$

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}, \quad w = \frac{D_4}{D}$$

$$\boxed{x = \frac{-2}{-2} = 1, \quad y = z = w = 0}$$

Review Test #2

Question 2

MTH 362

(A)
$$\begin{bmatrix} 0 & 0 & 3 & 1 \\ 0 & 1 & 4 & 2 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Cofactor exp col 1

$$0 - 0 + 1\begin{bmatrix} 0 & 3 & 1 \\ 1 & 4 & 2 \\ 1 & 0 & 1 \end{bmatrix} - 0$$

↳ cofactor exp row 3

$$= 1\left(1\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} - 0 + 1\begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix}\right)$$

$$= 1\left[1(6-4) - 0 + 1(0-3)\right]$$

$$= 1\left[2 - 3\right]$$

$$= \boxed{-1}$$

(B)

SWAP $R_1, R_3$

$$= -\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 1 & 0 & 1 \end{vmatrix}$$

$R_4 - R_2$ (NOT $R_2 - R_4$!)

$$= -\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & -4 & -1 \end{vmatrix}$$

$R_4 + \frac{4}{3} R_3$

$$= - \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1/3 \end{vmatrix}$$

<u>upper triangular</u>

$$= -\left[ 1 \times 1 \times 3 \times 1/3 \right]$$

$$= \boxed{-1}$$

✓

(3) $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 4 & 1 & 1 & 3 \\ 0 & 1 & 4 & 0 \end{bmatrix}$

• Find a row-echelon form & count pivots. •

$R_2 - 4R_1$

$$\sim \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -7 & -3 & 3 \\ 0 & 1 & 4 & 0 \end{vmatrix}$$

SWAP $R2$, $R3$

$$\sim \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & -7 & -3 & 3 \end{vmatrix}$$

$R_3 + 7R_2$

$$\sim \begin{vmatrix} \boxed{1} & 2 & 1 & 0 \\ 0 & \boxed{1} & 4 & 0 \\ 0 & 0 & \boxed{25} & 3 \end{vmatrix}$$

<u>3 Pivots</u> $\Rightarrow$ Rank $A = 3$

Added notes for 3
→ above and beyond what review asks for, but ~~may be~~ on test.

$\boxed{3 \text{ con't}}$

$$\text{Row } A = \text{span}\left( [1,2,1,0],\ [0,1,4,0],\ [0,0,25,3] \right)$$

or

$$\text{span}\left( [1,2,1,0],\ [0,-7,-3,3],\ [0,1,4,0] \right)$$

$$\text{col } A = \text{span}\left( \begin{bmatrix}1\\4\\0\end{bmatrix}, \begin{bmatrix}2\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\4\end{bmatrix} \right)$$

NOT

$$\text{span}\left( \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}1\\4\\25\end{bmatrix} \right)$$

Must use the pivot colums of the original matrix, not the reduced matrix!

4) $y' = \sqrt{4-y^2}\, x \quad , \quad y(0) = 0$

$$\int \frac{dy}{\sqrt{4-y^2}} = \int x\,dx$$

$$\arcsin\left(\frac{y}{2}\right) = \frac{x^2}{2} + C$$

$$y = 2\sin\left(\frac{x^2}{2} + c\right)$$

$$y(0) = 0$$

$$\Rightarrow 0 = 2\sin(0 + c)$$

$C = 0$

$$\boxed{y = 2\sin\left(\frac{x^2}{2}\right)}$$

5) $(4x^2y^3 - 2y)\,dx + \left(3x^3y^2 + \frac{2y}{x}\right)dy$

$$P\,dx + Q\,dy = 0$$

If the integrating Factor, F, depends on x – only
it satisfies

$$\frac{1}{F}F' = \frac{1}{Q}(P_y - Q_x) = R$$

$$P_y = 12x^2y^2 - 2$$
$$Q_x = 9x^2y^2 - \frac{2y}{x}$$

$$R = \frac{1}{Q}(P_y - Q_x)$$
$$= \frac{1}{3x^3y^2 + \frac{2y}{x}}\left(12x^2y^2 - 9x^2y^2 + \frac{2y}{x^2}\right)$$
$$= \frac{1}{3x^3y^2 + \frac{2y}{x}} \cdot \left(3x^2y^2 + \frac{2y}{x^2}\right)$$

$$\frac{1}{F}\cdot F' = \frac{1}{x}$$
$$\frac{d}{dx}(\ln|F|) = \frac{1}{x} = \frac{d}{dx}(\ln|x|)$$
$$|F| = |x| + c$$
$$F = x$$

Multiply by $F(x) = x$

$$(4x^3y^3 - 2x)\,dx + (3x^4y^2 + 2y)\,dy = 0$$
$$M\,dx + N\,dy = 0$$

$$\left[\begin{array}{ll} \text{Check} & M_y = 12x^3y^2 \\ & N_x = 12x^3y^2 \end{array}\right] \checkmark$$

$$u = \int M\,dx$$
$$= \int (4x^3y^3 - 2x)\,dx$$
$$= x^4y^3 - x + k(y)$$

$u_y = N$

$$\Rightarrow 3x^4y^2 - 0 + k'(y) = 3x^4y^2 + 2y$$

$$k'(y) = 2y$$
$$k(y) = y^2$$

Implicit sol.n

$$u(x,y) = C$$
$$\Rightarrow x^4y^3 - x^2 + y^2 = C$$

MTH 362 Question 6

Test 2 review

Final = dec 12
@ 11:30 - 2:30
in Tyler 109

6) Bernoulli

$$y' + 2xy = -xy^4$$

let $u = y^{1-4} = y^3$

$u' = -3y^{-4} \cdot y'$

Divide both sides by $y^4$

$$\frac{1}{y^4} y' + \frac{2x}{y^3} = -x$$

Recall:

$y' + P(x)y = r(x)$

has soln $y = e^{-h}\left(\int e^h r(x)\,dx + C\right)$

$h = \int p(x)\,dx$

$$-\frac{1}{3}u' + 2xu = -x$$

$$u' - 6xu = 3x$$

here

$p(x) = -6x$

$h(x) = \int p\,dx = -3x^2$ (no need for c)

$e^{h(x)} = e^{x^2}$

$u = e^{-h}\left(\int e^h r(x)\,dx + C\right)$

$u = e^{3x^2}\left(\int e^{-3x^2} \cdot 3x\,dx + c\right)$

$u = e^{3x^2}\left(-\frac{1}{2}e^{-3x^2} + c\right)$

$u = -\frac{1}{2} + ce^{3x^2}$

also $u = \frac{1}{y^3}$

$\hookrightarrow y = \frac{1}{\sqrt[3]{u}}$

$$= \boxed{y = \frac{1}{\sqrt[3]{-\frac{1}{2} + Ce^{3x^2}}}}$$