Math 362 Worksheet #2. Answers

1. Solve the IVP

$$y' = (1+y)\sin x, \quad y(0) = 1.$$

Ans. $y = 2e \cdot e^{-\cos x} = 2e^{1-\cos x}.$

2. Solve the inhomog. first order IVP

$$y' + 2xy = x, \quad y(0) = 2.$$

Ans $y = \frac{1}{2} + \frac{3}{2}e^{-x^2}.$

3. Use an integrating factor depending only on x to solve

$$(e^{xy} - y)dx + (xe^{xy} + 1)dy = 0$$

Ans $xe^y + ye^{-x} = C$

4. Solve the IVP

$$y'' + y' + 4y = 0 \qquad y(0) = 1, \; y'(0) = 0.$$

Ans. $y = e^{-x/2}\left(\cos\left(\frac{\sqrt{15}}{2}x\right) - \frac{1}{\sqrt{15}}\sin\left(\frac{\sqrt{15}}{2}x\right)\right)$

5. Use undetermined coeffs to find the general soln of the inhomog ODE

$$y'' - 4y' + 4y = 2e^{2x}.$$

Ans: $y = Ae^{2x} + Bxe^{2x} + x^2e^{2x}$

6. Use variation of parameters to solve.

$$y'' - 4y = e^x.$$

Ans. $y = c_1e^{2x} + c_2e^{-2x} - \frac{e^x}{3}.$

7. Find the general soln of the 4th order linear homog ODE

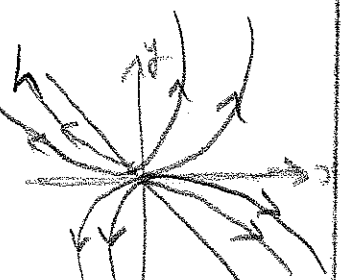
$$y^{IV} - 2y'' + y = 0.$$

Ans. $y = c_1e^{-x} + c_2xe^{-x} + c_3e^x + c_4xe^x$

3. Find all evalues & evectors of $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$.

Use this to specify the general soln of $\underline{y}' = A\underline{y}$ and make a rough sketch of the phase portrait.

$$y = c_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}e^x + c_2\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{2x}$$



Unstable Node.