MTH 362-02: Fall 2004
Final Review
You will be provided with the following during the Final Exam:

1. Table A7 and A8 Normal Distributions

2. The Table for Undetermined coefficients

Method of Undetermined Coefficients

Terms in $r(x)$	Choice for y_p
$ke^{\gamma x}$	$Ce^{\gamma x}$
$kx^n (n = 0, 1, \ldots)$	$K_n x^n + K_{n-1}x^{n-1} + \cdots + K_1 x + K_0$
$k \cos \omega x$	$K \cos \omega x + M \sin \omega x$
$k \sin \omega x$	$K \cos \omega x + M \sin \omega x$
$ke^{\alpha x} \cos \omega x$	$e^{\alpha x}(K \cos \omega x + M \sin \omega x)$
$ke^{\alpha x} \sin \omega x$	$e^{\alpha x}(K \cos \omega x + M \sin \omega x)$

3. The parameter formulas for the Method of variation of parameters:

$$u = -\int_{x_0}^{x} \frac{y_2 r(x)}{W(y_1, y_2)}$$
$$v = \int_{x_0}^{x} \frac{y_1 r(x)}{W(y_1, y_2)}$$

The rest you have to remember. The formulas you will need to remember are reflected in the questions below.

1. What is the polar form for $4 - i$?

2. What is the standard form of the complex number $-13.5(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}))$?

3. Given the following matrices

$$B = \begin{bmatrix} 4 & -12 \\ 1 & -3 \\ -3 & 9 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 3 \\ -4 & -1 \end{bmatrix}$$

   (a) Multiply matrices A and B, to get AB.

   (b) Does BA exist? Justify your answer.

   (c) Are the rows of matrix A linearly indenpendent? Justify your answer.

4. Compute the determinant of

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 5 \\ 6 & 8 & 0 \end{bmatrix}$$

5. Find the rank of the augmented matrix and discuss all solutions of the system:

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & x_3 & - & x_4 & = & 1 \\ x_1 & + & 2x_2 & - & 2x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & - & 3x_3 & - & x_4 & = & 1 \\ x_1 & + & 4x_2 & - & 4x_3 & - & x_4 & = & 1 \end{array}$$

6. (a) Sketch the probability function $f(x) = \frac{x^2}{30}(x = 1, 2, 3, 4)$ and the distribution function

   (b) Find the mean and the variance of the random variable X for the probability function in (a).

7. Suppose that 6% of steel rods made by a certain machine are defective, the defectives occurring at random during production. Find the probabilities that in a package of 100 stelel rods, the following numbers are defective.

   (a) 3 or fewer. Solve first by using the binomial distribution model and then by using the Poisson distribution.

   (b) exactly 11

8. The amount of time between taking a pain reliever and getting relief is normally distributed with a mean of 23 minutes and a standard deviation of 4 minutes. Find the probability that the time between taking the medication and getting relief is as follows.

   (a) at least 30 minutes

   (b) at most 20 minutes

9. Let X be normal with mean 10 and standard deviation 2. Determine c such that

   (a) $P(X \le c) = 95\%$

   (b) $P(X \le c) = 5\%$

   (c) $P(X \le c) = 99.5\%$

10. Solve the separable equation and the initial value problem

(a) $y' + 3x^2y = 0$

(b) $e^x y' = 2(x+1)y^2, \quad y(0) = \dfrac{1}{6}$

11. Solve the following linear differential equations and the Bernoulli equation.

(a) $y' + 2xy = 4x$

(b) $xy' = y^2 + y$ (use $y/x = u$)

(c) $y' + 3y = \sin x, \quad y(\frac{\pi}{2}) = 0.3$

(d) $y' + xy = xy^{-1}$

12. Show that $y_1(t) = \sqrt{t}$ and $y_2(t) = \dfrac{1}{t}$ are solutions of the differential equation

$$2t^2y'' + 3ty' - y = 0.$$

Show that the $y_1(t)$ and $y_2(t)$ are linearly independent solutions for the ODE.

13. Solve the following second order homogeneous equations

(a) $y'' + 4y' - 21y = 0$

(b) $y'' - 2y' + 2y = 0$

(c) $\begin{cases} y'' + 2y' + y = 0 \\ y(1) = 3, y'(1) = -3 \end{cases}$

14. A spring-mass-dashpot system consists of a mass $m = 10$ kg attached to a spring with spring constant $k = 100$ N/m; the dashpot has damping constant $c = 7$ kg/s. At time $t = 0$, the system is set into motion by pulling the mass down 0.5 m from equilibrium rest position while simultaneously giving it an initial downward velocity of 1 m/s.

(a) State the initial value problem to be solved for $y(t)$, the displacement from equilibrium (in meters) measured positive in the downward direction. Give numerical values to all constants involved.

(b) Solve the initial value problem. What is $\lim_{t\to\infty} y(t)$? Explain why your answer for this limit makes sense from a physical perspective.

15. Find the general solution of the following equations using variation of parameters:

(a) $y'' + 4y' + 4y = \dfrac{e^{-2x}}{x^3}$

(b) $y'' + y = \tan x, \quad 0 < x < \dfrac{\pi}{2}$

16. Find a particular solution of the following equation using the method of undetermined coefficients:

(a) $y'' + y' + y = x^3 + 2x^2$

(b) $3y'' + 10y' + 3y = 9x + 5\cos x$