

Math 362
Practice Exam I

1. Find the Cartesian and polar form of the reciprocal to the complex number $z = 3 - 4i$.
2. Find all cubic roots (in Cartesian and polar form) of $z = 8i$.
3. What is the polar form for $16 - 2i$? Find all values of $\ln(16 - 2i)$ and indicate the principal value $\text{Ln}(16 - 2i)$.
4. What is the standard form of the complex number $-13.5(\cos(0.58) + i\sin(0.58))$?
5. Find all the roots of $\sqrt[4]{-5}$.
6. Given $z = 1 - 2i$, what is e^z ?
7. Find all values of $(-2i)^{-i}$ and indicate the principal value.
8. For the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

find A^2, A^3, A^4 . Do you see a pattern?

9. Solve the system of algebraic equations by Gauss elimination.

$$\begin{cases} x + 3y - 4z = -2 \\ -2x - y + 2z = 6 \\ 4x - 6y + z = 9 \end{cases}$$

10. Use Gaussian elimination to obtain the solution of the following system of algebraic equations:

$$\begin{cases} 2x + 5y + 3z = 1 \\ -x + 2y + z = 2 \\ x + y + z = 0 \end{cases}$$

11. Rewrite the following system of algebraic equations

$$\begin{cases} x - 2y + 2z = -6 \\ -2x + y - 2z = 0 \\ 2x + 2y + z = 3 \end{cases}$$

in the matrix form $Ax = b$ and solve it by Gaussian elimination.

12. Write the following system in matrix form.

$$x + y + z = 1, x + 2x + 3z = 2, y + z = 3.$$

What are the coefficient matrix A and the augmented matrix \overline{A} ? What are their ranks? Solve this system.

13. For the following linear system determine all values of a for which the resulting linear system has (a) no solutions; (b) a unique solution; (c) infinitely many solutions.

$$x + y + z = 2, 2x + 3x + 2z = 5, 2x + 3y + (a^2 - 1)z = a + 1/.$$

14. Given the following matrices,

$$A = \begin{bmatrix} 4 & -12 \\ 1 & -3 \\ -3 & 9 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -4 & -1 \end{bmatrix}$$

- (a) Multiply matrices A and B to get AB .
- (b) Does BA exist? Justify your answer.
- (c) Are the columns of matrix A linearly independent? Justify your answer.
- (d) Find the rank of A , B , and AB .
- (e) Do the columns of matrix A span \mathbf{R}^3 ? Justify your answer.
- (f) Do the columns of matrix B span \mathbf{R}^2 ? Justify your answer.

15. Mark each statement True or False.

- (a) ___ In some cases, it is possible for six vectors to span \mathbf{R}^5 .
- (b) ___ If a system of linear equations has two different solutions, then it has infinitely many solutions.
- (c) ___ The equation $Ax = b$ is homogeneous if the zero vector is a solution.
- (d) ___ If v_1 and v_2 span a plane in \mathbf{R}^3 and if v_3 is not in that plane, then $\{v_1, v_2, v_3\}$ is a linearly independent set.