MTH 362 **Advanced Engineering Mathematics** **Name**_____
Test 2 **October 28, 2005**

DO ONLY SIX PROBLEMS. FOR EXTRA CREDIT DO SEVEN OR EIGHT PROBLEMS.
Note: You will not be given full credit if you only give the final answer. Show as much detail as you can.

1. Given the differential equation $x^2y'' - 3xy' + 4y = 0, x > 0$.
(a) (10 pts) Show that $y_1 = x^2$ and $y_2 = x^2 \ln x$ are solutions to the given ODE.

(b) (5 pts) Show that y_1 and y_2 are linearly independent solutions.

2. (20 pts) Solve the differential equation $xy' = x + y$ by substituting $u = \frac{y}{x}$.

3. Consider the equation $(4x^3y^3 - 2xy)dx + (3x^4y^2 - x^2 + 2y)dy = 0$.
(a) (5 pts) Show that the equation is exact.

(b) (15 pts) Solve for the general solution of the equation.

4. (15 pts) A differential equation and one of its solutions is given. Apply the method of reduction of order to obtain another linearly independent solution.

$$x^4y'' + 2x^3y' - y = 0, x > 0, y_1 = e^{1/x}$$

5. In a certain culture of bacteria the rate of increase is proportional to the number present.
(a) (10 pts) If there are 10^4 at the end of 3 hours and $4 \cdot 10^4$ at the end of 5 hours, how many were there at the beginning?

(b) (5 pts) If it is found that the number doubles every 4 hours, how many may be expected at the end of 12 hours.

6. An R-L circuit consists of a 100 volt DC battery connected in series with a 2 henry inductor and a 6 ohm resistor.
(a) (5 pts) Use Kirchoff's law to write the initial value problem assuming current starts to flow when the open switch is closed.

(b) (5 pts) Verify that $\frac{50}{3}\left(1 - e^{-3t}\right), t \geq 0$, is a solution to the IVP in part (a).

(c) (5 pts) At what time t does the current $I(t)$ reach 99% of its steady state value?

DO EITHER PROBLEM 7' OR PROBLEM 7".

7'. (15 pts) Solve the initial value problem $\begin{cases} y' + 4y = 20 \\ y(0) = 2 \end{cases}$.

7". (15 pts) Given the Bernoulli equation

$$\frac{dy}{dx} + 2xy = -xy^4$$

Use the substitution $v = y^{-3}$ to solve for its general solution. (Express your answer in terms of y.)