Math 215 Homework 2

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Due Friday June 3, 2016

Note: All problems taken from the fifth edition of the textbook. If you have the fourth edition, you may do the same numbered problems as these which have different numbers, but are otherwise the same.

1.7.2 Determine if the following set of vectors is linearly independent:

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}.$$

1.7.5,7 Determine if the columns of the matrices below form a linearly independent set.

$$\begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$$

Please turn over!

1.7.13 For what value of h is the following set of vectors linearly *dependent*? Justify your answer.

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

1.7.15,16,17,19 Determine by inspection whether each of the following sets of vectors is linearly *independent*. Justify each answer.

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -6 \\ 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} -8 \\ 12 \\ -4 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

Please turn over!

1.8.1 Let

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

and define $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find the images under T of

$$\mathbf{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}.$$

1.8.3,5 In each of the following with T defined by $T(\mathbf{x}) = A\mathbf{x}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$ and determine whether $\mathbf{x}$ is unique.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

1.8.9 Let

$$A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$$

and find all $\mathbf{x} \in \mathbb{R}^4$ which are mapped to $\mathbf{0}$ by the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

Please turn over!

1.8.16 (1.8.15 in 4th Ed.) Let

$$\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

and let T be defined by

$$T(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Use a rectangular coordinate system to plot $\mathbf{u}$ and $\mathbf{v}$ and their images under T. Describe geometrically what T does to each vector $\mathbf{x}$ in $\mathbb{R}^2$.

1.8.32 Show that the transformation T defined by $T(x_1, x_2) = (x_1 - 2|x_2|, x_1 - 4x_2)$ is not linear.

1.9.1,3,5,6,8 (1.9.1,3,6,10 in 4th Ed.) Find the standard matrix of each of the following linear transformations T

a. $T : \mathbb{R}^2 \mapsto \mathbb{R}^4$, $T(\mathbf{e}_1) = (3, 1, 3, 1)$ and $T(\mathbf{e}_2) = (-5, 2, 0, 0)$ where $\mathbf{e}_1 = (1, 0)$ and $\mathbf{e}_2 = (0, 1)$.

b. $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ rotates points about the origin through $3\pi/2$ radians (anti-clockwise).

c. $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is a vertical shear transformation maps $\mathbf{e}_1$ to $\mathbf{e}_1 - 2\mathbf{e}_2$, but leaves $\mathbf{e}_2$ unchanged.

d. $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ first reflects points through the horizontal x_1-axis and then reflects points in the line $x_2 = x_1$.

Please turn over!

1.9.19 Show that the transformation $T : \mathbb{R}^3 \mapsto \mathbb{R}^2$

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$$

is linear by finding a matrix which implements T.