Math 215 Homework 1

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1.7.2 Determine if the following set of vectors is linearly independent:

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}.$$

1.7.5,7 Determine if the columns of the matrices below form a linearly independent set.

$$\begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$$

1.7.13 For what value of h is the following set of vectors linearly *dependent*? Justify your answer.

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

Please turn over!

1.7.15,16,17,19 Determine by inspection whether each of the following sets of vectors is linearly *independent*. Justify each answer.

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} -8 \\ 12 \\ -4 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

1.8.1 Let

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

and define $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find the images under T of

$$\mathbf{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}.$$

Please turn over!

1.8.3,5 In each of the following with T defined by $T(\mathbf{x}) = A\mathbf{x}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$ and determine whether $\mathbf{x}$ is unique.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

1.8.9 Let

$$A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$$

and find all $\mathbf{x} \in \mathbb{R}^4$ which are mapped to $\mathbf{0}$ by the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

1.8.16 Let

$$\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

and let T be defined by

$$T(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Use a rectangular coordinate system to plot $\mathbf{u}$ and $\mathbf{v}$ and their images under T. Describe geometrically what T does to each vector $\mathbf{x}$ in $\mathbb{R}^2$.

Please turn over!

1.8.32 Show that the transformation T defined by $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$ is not linear.

1.9.1,3,6,8 Find the standard matrix of each of the following linear transformations T

a. $T : \mathbb{R}^2 \mapsto \mathbb{R}^4$, $T(\mathbf{e}_1) = (3, 1, 3, 1)$ and $T(\mathbf{e}_2) = (-5, 2, 0, 0)$ where $\mathbf{e}_1 = (1, 0)$ and $\mathbf{e}_2 = (0, 1)$.

b. $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ rotates points about the origin through $3\pi/2$ radians (anticlockwise).

c. $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is a vertical shear transformation which leaves $\mathbf{e}_1$ unchanged and maps $\mathbf{e}_2$ to $\mathbf{e}_2 + 3\mathbf{e}_1$.

d. $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ first reflects points through the horizontal x_1-axis and then reflects points in the line $x_2 = x_1$.

1.9.19 Show that the transformation $T : \mathbb{R}^3 \mapsto \mathbb{R}^2$

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$$

is linear by finding a matrix which implements T.