Taylor polynomials/series

MTH 142 Spr 2011 L. Pakula

We will use *Mathematica* to compute Taylor polynomials, and explore graphically and numerically how well they approximate functions. Start by defining a familiar function, like f(x) = sin(x):

In[1]:=
```
f[x_] := Sin[x]
```

Mathematica has a built-in command to compute Taylor polynomials called **Series** but the result is in a special format. To get our Taylor polynomials in a more useable form, we define a new function, **P**, as follows. We need to supply the order of the polynomial **n**, the name of the function, **f**, the point about which we are forming the Taylor polynomial, **a**, and the variable name **x**:

In[2]:=
```
P[n_, f_, a_, x_] := Normal[Series[f[t], {t, a, n}]] /. t → x
```

After evaluating this last cell and the one containing the definition of f(x), we can then get Taylor polyomials of our function f as in the next examples.

In[3]:=
```
P[5, f, 0, x]
```

Out[3]= $x - \frac{x^3}{6} + \frac{x^5}{120}$

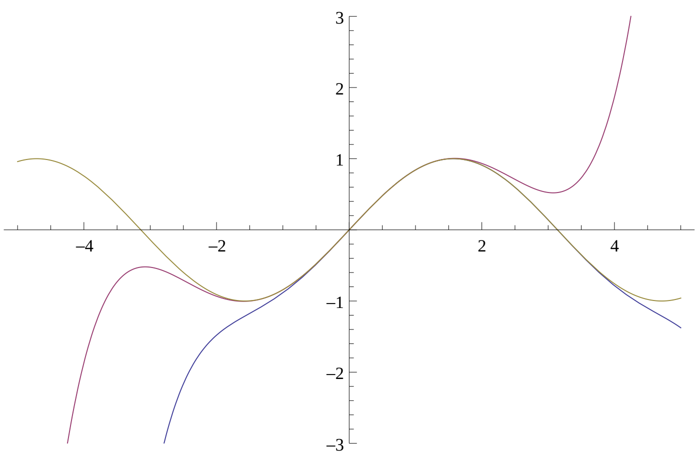
In[6]:=
```
P[6, f, Pi/2, x]
```

Out[6]= $1 - \frac{1}{2}\left(-\frac{\pi}{2} + x\right)^2 + \frac{1}{24}\left(-\frac{\pi}{2} + x\right)^4 - \frac{1}{720}\left(-\frac{\pi}{2} + x\right)^6$

Notice that when a = $\pi/2$, we get even powers of (x - a) = $\left(-\frac{\pi}{2} + x\right)$ rather than odd powers. Do you see why? Let's plot these polynomials together with f(x):

In[5]:=
```
Plot[{P[6, f, Pi/2, x], P[5, f, 0, x], f[x]}, {x, -5, 5}, PlotRange → {-3, 3}]
```

Out[5]=



Can you tell which graph is which? Since these polynomials have almost the same degree, the one about 0 should be a better approximation near x = 0, and the one about $\pi/2$ should be better near x = $\pi/2$. Suppose you want to get an approximate value for sin(0.5) (remember that's 0.5 RADIANS), using only the simple arithmetic involved in computing a Taylor polynomial of degree around 5 or 6. Let's compute the values of our two polynomials and compare it to *Mathematica*'s value for sin(0.5):

```
{P[6, f, Pi/2, 0.5], P[5, f, 0, 0.5], f[0.5]}
```

```
{0.479383, 0.479427, 0.479426}
```

We see that the 5th degree polynomial about 0 gives a closer answer than the 6th degree polynomial about $\pi/2$.

PROBLEMS

Start a new notebook, and begin with the commands:

```
f[x_] := Sin[x]
P[n_, f_, a_, x_] := Normal[Series[f[t], {t, a, n}]] /. t → x
```

Problem 1. Consider again the two Taylor polynomials for sin(x): the 5th degree polynomial about x = 0 and the 6th degree polynomial about $x = \pi/2$.

i) Which of these two polynomials gives a more accurate value for sin (0.9)? For sin (2.1)? If we changed the polynomial about 0 from degree 5 to degree 9 would that alter your answers? ii) Suppose you want to approximate sin (2.3). What is the smallest degree for the polynomial around 0 that will result in an approximation comparably accurate to the approximation of sin (2.3) using the 6th degree Taylor polynomial about $\pi/2$?

Problem 2. What order Taylor polynomial about 0 should you use to get fairly accurate approximations to sin(x) if x is between $-\pi/2$ and $\pi/2$? Between -5 and 5? Remember that a lower degree polynomial is less work to compute, even for a microprocessor.