Worksheet #7

1. **(a)** 2, 5, 10, 17, 26, 37, …..

Recursive formula: $\begin{cases} s_1 = 2 \\ s_n = s_{n-1} + (2n-1) \quad \text{for } n \geq 2 \end{cases}$

Explicit formula: $s_n = n^2 + 1$

(b) 1, –3, 5, –7, 9, –11, …..

Recursive formula: $\begin{cases} s_1 = 1 \\ s_n = (-1)^{n+1} \cdot \left(|s_{n-1}| + 2\right) \quad \text{for } n \geq 2 \end{cases}$

Explicit formula: $s_n = (-1)^{n+1} \cdot (2n-1)$

2. **(a)** $a_n = (-.3)^n$. Then $\{a_n\}$ is a geometric sequence with ratio – 0.3. This sequence converges and $\lim\limits_{n\to\infty} a_n = 0$.

(b) $a_n = \frac{n}{10} + \frac{10}{n}$. Then the sequence $\{a_n\}$ diverges.

(c) $a_n = \cos(\pi n)$. Then $\{a_n\} = -1, 1, -1, 1, -1, 1, \ldots..$
So, $\{a_n\}$ diverges.

(d) $a_n = \frac{2n+(-1)^n \cdot 5}{4n-(-1)^n \cdot 3}$. As n gets larger and larger, the value of $\frac{2n+(-1)^n \cdot 5}{4n-(-1)^n \cdot 3}$ gets closer and closer to the value of $\frac{2n}{4n}$. So, $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{2n+(-1)^n \cdot 5}{4n-(-1)^n \cdot 3} = \lim\limits_{n\to\infty} \frac{2n}{4n} = \frac{1}{2}$.

(e) $a_n = \frac{\sin n}{n}$. $-1 \leq \sin n \leq 1$ for every n. Thus, $\frac{-1}{n} \leq \frac{\sin n}{n} \leq \frac{1}{n}$ for every $n \neq 0$. Since $\lim\limits_{n\to\infty} \frac{1}{n} = 0$, and $\lim\limits_{n\to\infty} \frac{-1}{n} = 0$, the limit of $\frac{\sin n}{n}$ must also equal 0 (as $n \to \infty$).

3. **(a)** $\sum_{n=0}^{10} 7(3)^n = 7 \cdot \frac{1-3^{10+1}}{1-3} = 620{,}011$

(b)

$$\sum_{n=2}^{5} 2(-4)^n = 2 \cdot \left(\sum_{n=0}^{5} (-4)^n - \sum_{n=0}^{1} (-4)^n \right) = 2 \cdot \left(\frac{1-(-4)^{5+1}}{1-(-4)} - \frac{1-(-4)^{1+1}}{1-(-4)} \right) = -1632$$

(c) $\sum_{n=0}^{\infty} z \cdot y^n$, $|y| < 1$. Since $|y| < 1$, $\sum_{n=0}^{\infty} z \cdot y^n = z \cdot \frac{1}{1-y} = \frac{z}{1-y}$.

(d) $\sum_{n=0}^{\infty} 3(4)^n$ diverges since $|4| \geq 1$.

4. **(a)** $\sum_{n=1}^{\infty} \frac{1}{e^n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$. Since $\left|\frac{1}{e}\right| < 1$, this is a geometric series that will converge.

However, the instructions were to use the integral test.

$$\int_1^{\infty} \frac{1}{e^x}\, dx = \lim_{n \to \infty} \int_1^n e^{-x}\, dx = \lim_{n \to \infty} \left(-e^{-x} \Big|_{x=1}^{x=n} \right) = \lim_{n \to \infty} \left(-e^{-n} + e^{-1} \right) = 0 + e^{-1} = \frac{1}{e}.$$

So the series $\sum_{n=1}^{\infty} \frac{1}{e^n}$ converges. In fact, $\sum_{n=1}^{\infty} \frac{1}{e^n} = \frac{1}{e-1}$.

(b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$.

$$\int_2^{\infty} \frac{1}{x(\ln x)}\, dx = \lim_{n \to \infty} \int_2^n \frac{1}{x(\ln x)}\, dx = \lim_{n \to \infty} \left(\ln(\ln(x)) \Big|_{x=2}^{x=n} \right) = \infty.$$

So the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$ diverges.

5. $\sum_{n=1}^{\infty} \frac{n}{n+1}$. Since $\lim_{n \to \infty} \frac{n}{n+1} = 1 \neq 0$, the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ must diverge.

for Dr. Comerford, MTH 142
Solutions by Chris Lynd