MATH 451/URI

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Exam #2

1. Given the values of the joint probability distribution of X and Y shown in the table:

		x		
		0	2	4
y	1	$\frac{1}{7}$	$\frac{1}{28}$	$\frac{2}{7}$
	2	$\frac{1}{7}$	$\frac{1}{28}$	$\frac{1}{14}$
	3	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{7}$

find

(a) the marginal distribution of X.

(b) the marginal distribution of Y.

(c) the conditional expectation of Y given $X = 1$, that is $E(Y \mid X = 1)$.

(d) $P(X + Y \geq 5)$.

2. If X is the amount of money (in dollars) that a salesperson spends on gasoline during a day and Y is the corresponding amount of money (in dollars) for which he or she is reimbursed, the joint density of these two random variables is given by

$$f(x,y) = \begin{cases} \frac{1}{25}\frac{20-x}{x} & \text{for} \quad 10 < x < 20, \frac{x}{2} < y < x \\ 0 & \text{otherwise} \end{cases}$$

find

(a) the marginal density of X.

(b) the expectation of Y.

(c) the conditional expectation of Y given $X = 16$,that is $E(Y \mid X = 16)$.

Note: In this problem we will consider X and Y to be continuous random variables.

3. For the random variables in Problem 1. check if they are independent and find

(a) the expectation EX, and variance VX.

(b) the expectation $E(XY)$ and $V(X + Y)$.

4. Let X and Y be continuous random variables given in Problem 2. Find

(a) the expectation EY and VY.

(b) For a given positive number a find $P(|X - EX| \geq a)$ exactly and using the Chebyshev inequality.

5. A certain brand of cereal has an advertised weight of 10 ounces, but the weights are actually random, with expected value 10.52 ounces and standard deviation .30 ounce. The manufacturer wants to know an interval of weights that will contain 90% of the weights of this cereal.

(a) If we can assume that the weights are normally distributed , then there are many such intervals. Find the one centered at 10.52.

(b) If we can assume that the weights are normally distributed, find such interval centered at 10.00.

(c) If we can assume that the weights are exponentially distributed with expected value 10.52 ounces, find such interval centered at 10.40.

(d) Suppose we can not assume anything about the distribution of weights, other than the given expected value and standard deviation. Find an interval that is sure to contain 90% of the weights.

(*Hint:* Use Chebyshev's inequality).