Answers

Math 107 Sec. 2
Montgomery
February 26, 2007 Exam 2

1. (20 pts.) U is the set of all 26 letters in the alphabet.
A = {a, e, i, o, u}, B = {a, b, c, … ,r}, C={h,i,l,a,r,y}.

a) (C ∪ B')' $B' = \{s,t,u,v,w,x,y,z\}$, $C \cup B' = \{h,i,l,a,r,s,t,u,\ldots,z\}$

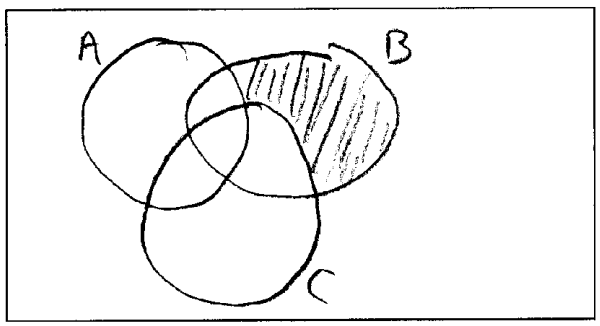
$(C \cup B')' = \{b,c,d,e,f,g,j,k,m,n,o,p,q\}$

b) B' ∩ (A ∪ C) $A \cup C = \{h,i,l,a,r,y,e,o,u,\}$

$B' \cap (A \cup C) = \{y, u\}$

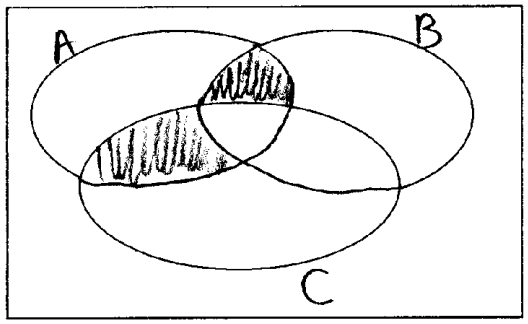
2. (10 pts) In a survey, students were asked if they enjoyed aerobics, if they enjoyed badminton, and if they enjoyed canoeing.

a. Draw a Venn diagram that could be used to represent the results of the survey, and shade the area that would represent the students who enjoyed badminton but not aerobics or canoeing.



$B \cap A' \cap C'$

b. What people are represented by the shaded area below. (Describe in words and in symbols)



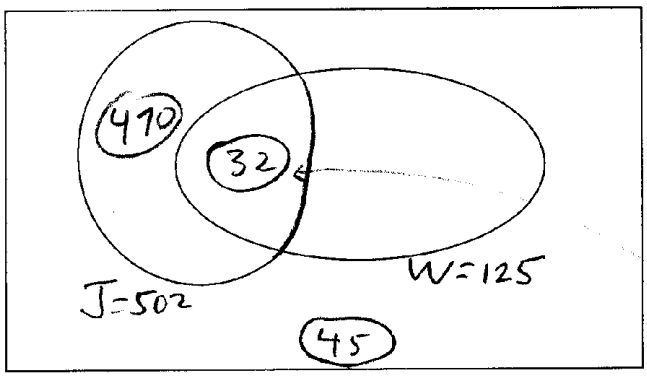
Those who enjoyed aerobics and badminton but not canoeing or enjoyed Aerobics + canoeing but not badminton

$(A \cap B \cap C') \cup (A \cap C \cap B')$

3. (15 pts) 640 US citizens earned a Ph. D. in Mathematics. 502 got jobs at universities and 125 of the 640 were women. 45 men who got Ph.D.'s did not get a university job. Use a Venn diagram to find:

a. How many women Ph. D.'s got university jobs? 320 ①

b. How many men got university jobs? 470



U=640

640-45=595 = n(J∪W)

502+125=627

627
595
32

② 502-32 = 470

5. (15 pts) A die is rolled twice, and two cards are dealt from a standard deck. The result is recorded for each.

a. How many outcomes are possible? (Calculate)

6 × 6 × 52 × 51 = 95,472

b. How many outcomes have the die rolls both even numbers, and the cards with different suits?
(For example, (2, 2, JH, JS) or (4, 6, 2D, AS))

3 × 3 × 52 × 39 = 18,252

c. How many outcomes have at least one of the die rolls being an odd number or the cards having the same suit (i.e., for how many outcomes does the description in part b) not hold)?

95,472
− 18,252
77,220

5. (20 pts) Show factors, cancel and compute:

a. ${}_{31}C_4 = \dfrac{31 \cancel{30}^{5}\ 29\ \cancel{28}^{7}}{\cancel{4}\ \cancel{3}\ \cancel{2}\ 1} = 31 \times 5 \times 29 \times 7 = 31{,}465$

b. ${}_{10}P_3 = 10 \cdot 9 \cdot 8 = 720$

c. ${}_{13}C_9 \times {}_8C_6 = \dfrac{13\ 12\ 11\ \cancel{10}^{5}\ \cancel{9}\ \cancel{8}\ 7\ \cancel{6}\ \cancel{5}}{\cancel{9}\ \cancel{8}\ \cancel{7}\ \cancel{6}\ \cancel{5}\ \cancel{4}\ \cancel{3}\ \cancel{2}\ 1} \times \dfrac{\cancel{8}^{4}\ \cancel{7}\ \cancel{6}\ \cancel{5}\ \cancel{4}\ \cancel{3}}{\cancel{6}\ \cancel{5}\ \cancel{4}\ \cancel{3}\ \cancel{2}\ 1}$

$= 13 \times 11 \times 5 \times 4 \times 7 = 20{,}020$

6. (20 pts) Julia is inviting 80 guests to her wedding reception. The two tables in front are for "honored guests". One has seating for 14, and the other for 8.

a. In how many ways can she choose the two groups to sit at these tables? (You don't need to compute the answer.)

$\dfrac{{}_{80}C_{14}}{\text{large table}} \times \dfrac{{}_{66}C_{8}}{\text{small table}}$ OR $\dfrac{{}_{80}C_{8}}{\text{small table}} \times \dfrac{{}_{72}C_{14}}{\text{large table}}$

b. Of the 80 guests, 43 are women, and 37 are men. In how many ways can she choose the two groups to sit at the tables if the larger table must contain all women, and the smaller table must contain half women, half men. (You don't need to compute the answer.)

$\dfrac{{}_{43}C_{14}}{\text{women at large table}} \times \dfrac{{}_{29}C_{4}}{\text{women at small table}} \times {}_{37}C_{4}$ (men at small table)