Answers

Math 107 Sec 1, 3
Montgomery
Feb 7, 2005 Test 1

1. Give the negative of the statement below. Use conventional English, without phrases like "It is not the case that…" or "It is false that…".

a. "Some boxers don't respect Mohammed Ali."

All boxers Respect Mohammed Ali.

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b. "The parking lot is too small and needs plowing."

The parking lot is not too small, or it does not need plowing.

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2. Translate into symbolic form. Then write **both in symbolic form and in conventional English** the converse, inverse, and contrapositive of the statement

If the Pats win, Brady goes to Disneyland.

P: the Pats win
Q: Brady goes To Disneyland

SYMBOLIC FORM: $P \to Q$

CONVERSE: $Q \to P$ If Brady goes To disneyland, the Pats won.

INVERSE: $\sim P \to \sim Q$ If The Pats don't win, Brady won't go To Disneyland

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CONTRAPOSITIVE: $\sim Q \to \sim P$ If Brady doesn't go to Disneyland, then the Pats didn't win

Which pairs of these statements are logically equivalent to one another?

Original $\equiv$ contrapositive, $\neq$ converse $\equiv$ inverse

3. Draw a Venn diagram representing the argument below, and use it to decide whether it is valid or invalid.

All pineapples are grown in Hawaii.
<u>This fruit was grown in Hawaii</u>.
Therefore, this fruit is a pineapple.

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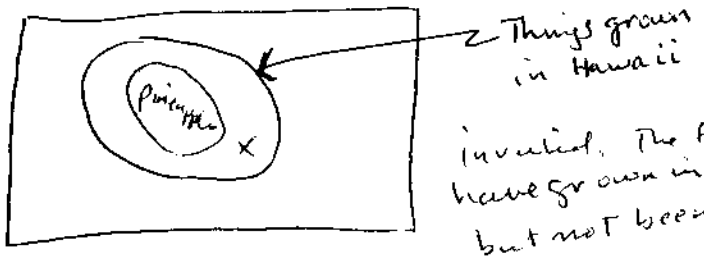


invalid. The fruit could have grown in Hawaii but not been a pineapple.

4. Find a set of values of p, q, and r which make the statement below false:

$$(\sim p \to q) \vee r$$

"$\vee$" is F when both are false so $r = F$ and

OR

$\sim p \to q = F$

T F

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$\sim p = T$ $\quad p = F$

$q = F$

| p | q | r | ~p | ~p → q | (~p → q) ∨ r |
|---|---|---|---|---|---|
| T | T | T | F | | |
| T | T | F | F | | |
| T | F | T | F | | |
| T | F | F | F | | |
| F | T | T | T | | |
| F | T | F | T | | |
| F | F | T | T | F | |
| F | F | F | T | F | F |

5. Fill in the blanks:

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a. $\sim(p \vee q)$ is equivalent to $(\sim p) \wedge (\sim q)$

b. $p \to q$ is false when $p = T$ and $q = F$

**For c. and d. below, write symbolically, using p: It is snowing. q: The streets are slippery.**

c. The streets are slippery whenever it snows. $p \to q$

d. The streets are slippery only if it snows. $q \to p$

e. An argument with hypotheses H1 and H2 and conclusion C is valid when $H1 \wedge H2 \to C$ is always T.

6. Translate the following argument into symbolic form, and use a truth table to determine if the argument is valid or not. You only need to show the top row and the next two rows. How will you know from the truth table that the argument is valid or not?

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If I take the bus it's because parking is inconvenient.
If we pay a fee, then parking will be convenient.

Therefore, if we pay a fee, I won't use the bus.

a) Symbolic form:

B: I take the Bus
P: Parking is convenient
F: we pay a fee

$B \to \sim P$

$F \to P$

$F \to \sim B$

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b) Truth table (top row and next two rows):

| B | P | F | ~P | ~B | B → ~P (H1) | F → P | F → ~B (C) | H1 ∧ H2 | H1 ∧ H2 → C |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | T | F | F | T |
| T | T | F | F | F | F | T | T | F | T |

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c) How will you know if it is valid?

If the last column $H1 \wedge H2 \to C$ is always T.

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BONUS: How can you tell without using a truth table if the argument above is valid?

$B \to \sim P$

$F \to P$ replace w. contrapositive $\sim P \to \sim F$

$F \to \sim B$

$B \to \sim P \to \sim F$

Transitivity

Valid.