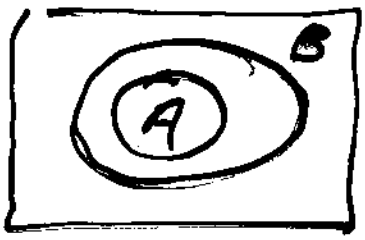
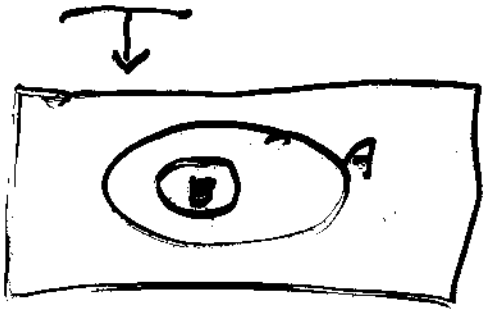
1. **Draw a Venn diagram which represents the situation where $A \cap B = A$. Also where $A \cup B = A$**

$A \cap B = A$



$A \cup B = A$



2. **(15 pts.) U is a standard deck of cards. A = {x ε U | x is a 10, a face card, or an ace}, B = is the set of all black cards in U, C={5s, 5h, 5d, 5c, 10s, 10h, 10d, 10c}**

a) **A ∩ B'**

B' = red cards

$A \cap B' = \{10h, 10d, Jh, Jd, \ldots Ah, Ad.\}$

(Both red and 10-A)

b) **$(C \cup A)'$**

$C \cup A$ = 5's, 10's, J → A

$(C \cup A)'$ = all 2, 3, 4 6, 7, 8, 9

c) **B ∩ (A ∪ C')**

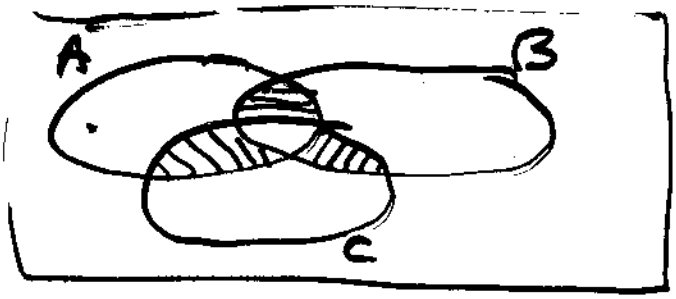
$A \cup C'$ = 10-A or not 5's or 10's

= everything but 5's.

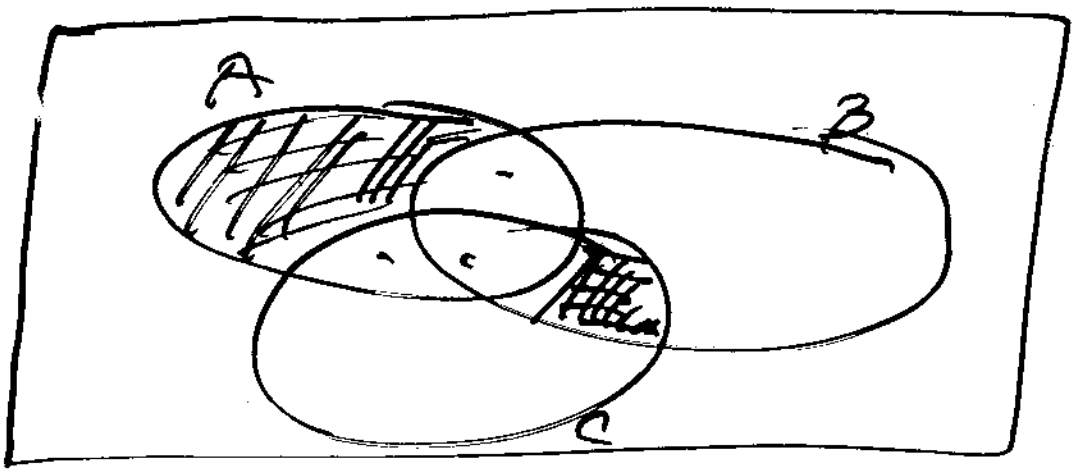
C' = all 2-4, 6-9, J-A

$B \cap (A \cup C') = \{$black and not 5$\}$

3. (15 pts) In a survey, students were asked if they liked apples, if they liked bananas, and if they liked cherries.
a. Draw a Venn diagram that could be used to represent the results of the survey, and shade the area that would represent the students who liked exactly two of the fruits.



b. What people are represented by the shaded area below.

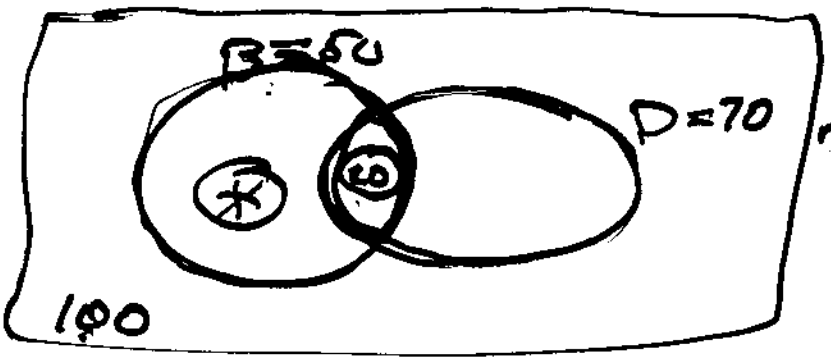


Those who like apples but not bananas or cherries, (OR) who like bananas and cherries but not apples

$(A \cap B' \cap C') \cup (B \cap C \cap A')$

4. (15 pts) 200 teenagers were given a survey that asked two questions: Are you satisfied with your body image? Do you have an eating disorder? 150 said they were not satisfied with their body image. 70 said they had an eating disorder, and 100 of those who said they were not satisfied with body image also said they had no eating disorder. Use a Venn diagram to find:

a. What proportion of the teenagers said they were satisfied with their body image and had an eating disorder? 1/10



$n = 200$

$n(B \cup D) = 200 - 100 = 100$

$50 + 70 - 100 = 20$

$\frac{20}{200}$

b. What proportion said they were satisfied with their body image and didn't have an eating disorder? 3/20

$50 - 20 = 30$

$\frac{30}{200} = \frac{3}{20}$

5. (15 pts) The standard RI license plate has two letters followed by 3 digits.

a. How many of these plates could there be?

$26 \times 26 \times 10 \times 10 \times 10$

b. If the two letters must be different, and the 3 digits must be different, how many plates?

$26 \times 25 \times 10 \times 9 \times 8$

c. How many plates start with the letters IQ and end with a number 130 or larger?

$1 \quad 1 \quad - \quad - \quad -$

999

$-\ 129$

( )

6. (15 pts) In how many ways can the nine different positions on a baseball team be filled from a pool of 23 children? Discuss whether order is important.

$23\,P_9$

$= 23 \cdot 22 \cdot 21 \cdots$ (9 factors)

$$\frac{10!}{(10-4)!\,4!} = \frac{10!}{6!\,4!} = \frac{10987\,6!}{4!\,6!}$$

7. **Write out the factors and compute:** ${}_{12}P_4 =$_____

${}_{10}C_4 =$_____

$${}_{12}P_4 = 12 \cdot 11 \cdot 10 \cdot 9 = \ldots.$$

$${}_{10}C_4 = \frac{10^{3}\cancel{9}\cancel{8}7}{\cancel{4}\cancel{3}\cancel{2}1} = 10 \cdot 3 \cdot 7 = 210$$

8. a) **How many 3 card hands can be formed from a standard 52 card deck?** ↳ no order

$${}_{52}C_3$$

b) **How many of these hands consist entirely of face cards?**

$${}_{12}C_3$$

c. **How many of these hands contain two aces and a king?**

$$\underbrace{{}_4C_2}_{\text{Aces}} \times \underbrace{{}_4C_1}_{\text{King}}$$

d. **How many ways are there to win second prize in a 6/45 lottery?**

$$\underbrace{{}_6C_5}_{\text{5 winners}} \times \underbrace{{}_{39}C_1}_{\text{1 loser}}$$