Math 132
Montgomery
October 1, 2003

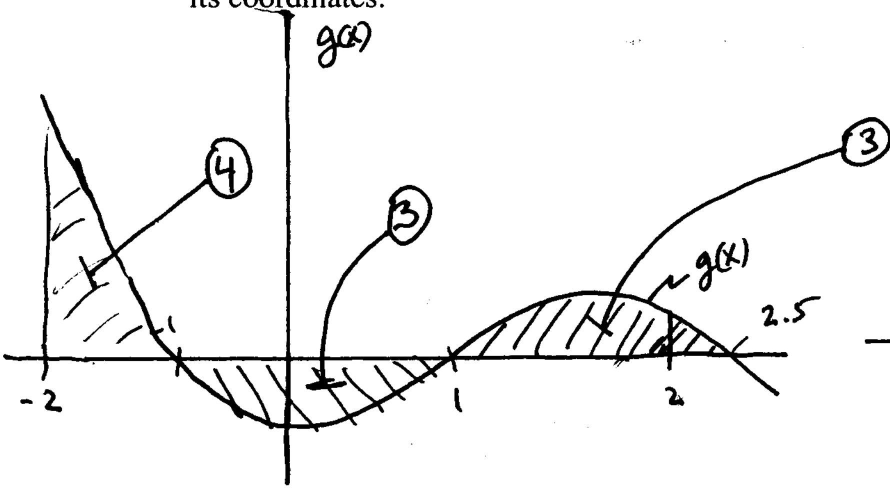
Exam I

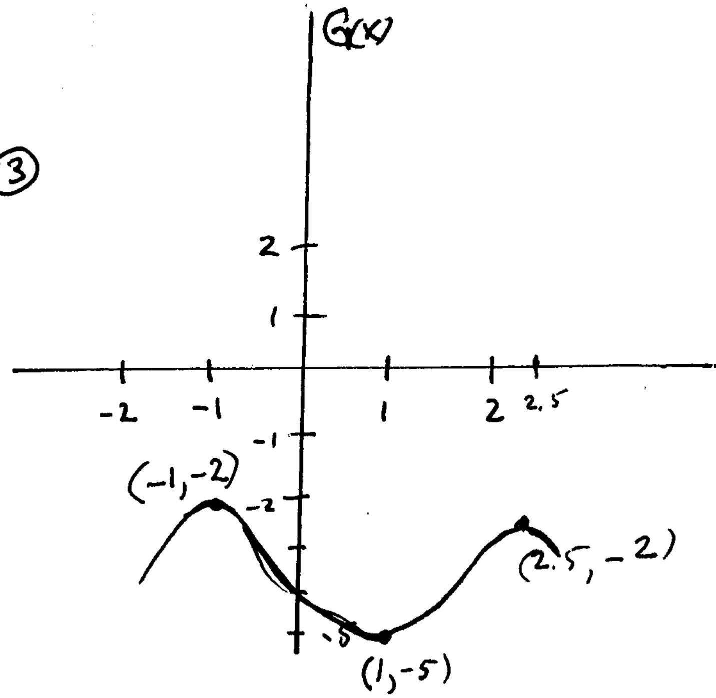
Answers

1. a) (7 pts) A helicopter is initially 100 feet above the ground and begins to gain altitude at a rate of $f(t) = \ln(1+t^2)$ feet per second. How high is the helicopter at the end of 5 minutes (300 secs)?

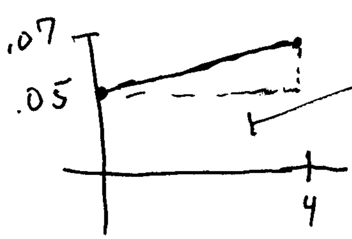
$$100 + \int_0^{300} \ln(1+t^2)\,dt = 2925.41$$

b) (7 pts) The graph below is the derivative $g(x)$ of a function $G(x)$, where $G(-1) = -2$. Sketch the graph of $G(x)$ and label each critical point of $G(x)$ with both of its coordinates.





2. (9 pts) The relative growth rate of a population starts at 5% and increases linearly to 7% after 4 years. By what percentage does the population increase or decrease over the 4 year period?



area $= A = .06 \times 4 = .24$

$$e^{.24} = 1.27 = \frac{P(4)}{P(0)}$$

27% increase

3. Find an antiderivative, use any method:

a. (7 pts) $Z(q)=\sqrt[3]{q^2}-\frac{5}{q}-\frac{1}{2q^2}$

$$\int q^{2/3} - 5\int \frac{1}{q} - \frac{1}{2}\int q^{-2}$$

$$= \frac{q^{5/3}}{5/3} - 5\ln|q| - \frac{1}{2}\frac{q^{-1}}{-1} + C = \frac{3}{5}q^{5/3} - 5\ln|q| + \frac{1}{2q} + C$$

b. (7 pts) $f(t)=\frac{3}{2\sqrt{t}}+\cos 5t$

$$= \frac{3}{2}\int t^{-1/2} + \int \cos 5t = \frac{3}{2}\frac{t^{+\frac{1}{2}}}{\frac{1}{2}} + \frac{\sin 5t}{5} + C$$

$$= 3\sqrt{t} + \frac{1}{5}\sin 5t + C$$

4. Find the integrals, use any method:

a. (7 pts) $\int \frac{2}{(3x+4)}dx = \frac{1}{3}2\int \frac{1}{u}\,du = \frac{2}{3}\ln|u| + C$

$u = 3x+4$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$= \frac{2}{3}\ln|3x+4| + C$$

b. (7 pts) $\int e^x\sqrt{1+2e^x}\,dx = \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} = \frac{1}{3}(1+2e^x)^{3/2} + C$

$u = 1+2e^x$

$du = 2e^x dx$

$\frac{1}{2}du = e^x dx$

c. (7 pts) $\int \frac{\sin x}{(1+\cos x)^3}dx = -\int \frac{du}{u^3} = -\int u^{-3}du$

$$= -\frac{u^{-2}}{-2} + C$$

$u = 1+\cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$= \frac{(1+\cos x)^{-2}}{2} + C$$

5. Use the method of substitution, IF POSSIBLE, to find the following definite integrals (if it is not possible to find the integrals by substitution, say so, and find the answer with your calculator instead).

a. (7 pts) $\int_0^4 \frac{1}{\sqrt{x}}\left(1+\sqrt{x}\right)^2 dx$ $= 2\int_{x=0}^{x=4} u^2\, du = \frac{2u^3}{3}\Big|_{\substack{x=0\\u=1}}^{\substack{x=4\\u=3}}$

$u = 1 + x^{\frac{1}{2}}$

$du = \frac{1}{2} x^{-\frac{1}{2}} dx$

$2du = \frac{1}{\sqrt{x}} dx$

$= \frac{2}{3}\left[3^3 - 1^3\right] = \frac{2}{3}(26)$

$= 52/3$

OR

$\frac{2}{3}\left(1+\sqrt{x}\right)^3\Big|_0^4 = 2(3^3-1^3) = \frac{52}{3} = 17\frac{1}{3}$

b. (7 pts) $\int_0^2 (\sin x)e^{\sin x}\, dx$.

$u = \sin x$

$du = \cos x\, dx$

Can't do with substitution

$\text{FnInt}(\sin x\, e^{\sin x}, x, 0, 2)$

$= 3.342$

6. The height x, in feet, of a certain species of tree has a density function given by $p(x) = Cxe^{-0.1x}$. The tree is never more than 35 feet high.

a. (7 pts) What is the value of C so that *p(x)* is a density function?

$$\int_0^{35} C x e^{-.1x} dx = C \int_0^{35} x e^{-.1x} dx = C \cdot 86.41 = 1$$

$$C = \frac{1}{86.41} = .0116$$

b. (7 pts) What fraction of trees of this species is over 20 feet high?

$$.0116 \int_{20}^{35} x e^{-.1x} dx = .3126$$

7. The density function for the duration of a telephone call is approximated by $p(x) = .4e^{-0.4x}$, where x is the duration of the call in minutes.

a. (7 pts) What is a formula for the cumulative distribution function for x?

$$P(t) = \int_0^t .4e^{-.4x} dx = \left. \frac{.4e^{-.4x}}{-.4} \right|_0^t = -e^{-.4t} + e^0$$

$$= 1 - e^{-.4t}$$

b. (7 pts) Use the cumulative distribution function to find the probability that a given call will last between 1 and 3 minutes.

$$P(3) - P(1) = \left(1 - e^{-.4 \cdot 3}\right) - \left(1 - e^{-.4 \cdot 1}\right)$$

$$= e^{-.4} - e^{-1.2} = .37$$