ANSWERS

MATH 215

Practice - Eigenvalues and Eigenvectors

Compute the eigenvalues and determine if the matrices are diagonalizable.

$$C = \begin{bmatrix} 1 & 6 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$p(\lambda) = \det(C - \lambda I) = 0$$

$$= \det \begin{bmatrix} 1-\lambda & 6 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{bmatrix} = 0$$

$$= (1-\lambda)\cdot \det \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$= (1-\lambda)\cdot\left((2-\lambda)(2-\lambda)-1\right) = 0$$

$$= (1-\lambda)\cdot(\lambda^2-4\lambda+3) = 0$$

$$= (1-\lambda)\cdot(\lambda-3)\cdot(\lambda-1) = 0$$

$\lambda=1, \quad \lambda=3, \quad \lambda=1$

Problem: check $\lambda=1$

$$(C - 1\lambda I) = \begin{bmatrix} 0 & 6 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\overset{rref}{\sim} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Has only 1 free variable and therefore only 1 eigenvector for $\lambda=1$

geo. mult $=1$ < alg. mult $=2$

Not diagonalizable

$$D = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$$p(\lambda) = \det(D - \lambda I) = 0$$

$$= \det \begin{pmatrix} 4-\lambda & 0 & -2 \\ 2 & 5-\lambda & 4 \\ 0 & 0 & 5-\lambda \end{pmatrix} = 0$$

$$= (5-\lambda)\cdot \det \begin{bmatrix} 4-\lambda & 0 \\ 2 & 5-\lambda \end{bmatrix} = 0$$

$$= (5-\lambda)\cdot(4-\lambda)\cdot(5-\lambda) = 0$$

$\lambda=5, \quad \lambda=5, \quad \lambda=4$

Problem: check $\lambda=5$

$$(D - 5I) = \begin{bmatrix} -1 & 0 & -2 \\ 2 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\overset{rref}{\sim} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Has 2 free variables and therefore has 2 Linear Ind. eigenvectors for $\lambda=5$. $\lambda=4$ has one ~~two~~ eigenvector and since $\lambda=4 \neq 5$ is linearly ind ~~to~~ eigenvectors for $\lambda=5$.

Therefore, D is diagonalizable.