

MATH 215
Practice 2.1

Given the linear systems of equations.

$$\begin{aligned}x_1 + 2x_2 + 4x_3 &= 1 \\ -2x_1 - 3x_2 - 5x_3 &= 0 \\ 2x_1 + x_2 &= -2\end{aligned}$$

1. Write the linear system as $Ax = b$.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & -3 & -5 \\ 2 & 1 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$Ax = b \Rightarrow \begin{bmatrix} 1 & 2 & 4 \\ -2 & -3 & -5 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

2. Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ -2 & -3 & -5 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 4 & 2 \\ 0 & 1 & 0 & -10 & -8 & -3 \\ 0 & 0 & 1 & 4 & 3 & 1 \end{array} \right] \Rightarrow A^{-1} = \begin{bmatrix} 5 & 4 & 2 \\ -10 & -8 & -3 \\ 4 & 3 & 1 \end{bmatrix}$$

3. Use A^{-1} to solve $Ax = b$

$$\begin{aligned}Ax &= b \\ A^{-1}(Ax) &= A^{-1}b \\ Ix &= A^{-1}b \\ x &= A^{-1}b\end{aligned} \quad X = \begin{bmatrix} 5 & 4 & 2 \\ -10 & -8 & -3 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$$