1. Let $S = \{(a - b, b - a) \mid a$ and b are real numbers $\}$ be a subset of R^2. Show that S is a subspace of R^2. Find the dimension of S.

2. Let $H = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : 3a - 2b = 5c \right\}$. Show H is a subspace of $\mathbf{R^3}$ and find the dimension of H

3. Assume that $A = [a_1, a_2, a_3, a_4, a_5]$ and $B = [b_1, b_2, b_3, b_4, b_5]$ are row equivalent (i.e. $B = \text{rref}(A)$), where

$$A = \begin{pmatrix} 1 & 2 & -2 & 0 & 7 \\ -2 & -3 & 1 & -1 & -5 \\ -3 & -4 & 0 & -2 & -3 \\ 3 & 6 & -6 & 5 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Find a basis for the column space of A.

(b) Find a basis for the row space of A.

(c) Find a basis for the null space of A.

(d) Find the $\dim(\text{Nul}(A))$ and $\dim(\text{Col}(A))$.

4. Find the eigenvalues for all matrices and eigenvectors for matrix A. Determine which matrices are diagonalizable.

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 6 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$