MATH 215
Practice

1. What must a transformation T from $\mathbf{R}^m$ to $\mathbf{R}^n$ satisfy in order to be a linear transformation?

① $T(u+v) = T(u) + T(v)$ for all u, v in domain of T

② $T(cu) = c\,T(u)$ for all scalars c and all u in domain of T

2. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation, such that $T([1,0]) = [2,1,3]$ and $T([0,1]) = [1,0,-2]$, find $T([2,3])$.

$$T\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = 2 \cdot T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) + 3\,T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = 2 \cdot \begin{bmatrix}2\\1\\3\end{bmatrix} + 3\begin{bmatrix}1\\0\\-2\end{bmatrix}$$

$$= \begin{bmatrix}4\\2\\6\end{bmatrix} + \begin{bmatrix}3\\0\\-6\end{bmatrix}$$

$$= \begin{bmatrix}7\\2\\0\end{bmatrix}$$

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation such that $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\4\\0\end{bmatrix}$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\3\\-1\end{bmatrix}$. Find the standard matrix representation of T.

$$A = \begin{bmatrix}T(e_1) & T(e_2)\end{bmatrix} = \begin{bmatrix}1 & -1\\4 & 3\\0 & -1\end{bmatrix}$$