3 Determinants and Cofactor Expansion

When we calculate the determinant of an $n \times n$ matrix using cofactor expansion we must find n $(n-1) \times (n-1)$ determinants. So (roughly) $C_n \approx nC_{n-1}$, where C_n is the complexity of finding an $n \times n$ determinant. Now $C_2 = 2$ (two multiplications).

n	2	3	4	5	...
C_n	2	$3 \cdot 2$	$4 \cdot 3 \cdot 2$	$5 \cdot 4 \cdot 3 \cdot 2$	...

We can see that $\boxed{C_n = n!}$

Given an $n \times n$ determinant to calculate, we may either use the cofactor method, with a runtime of $O(n!)$, or we may reduce the matrix using Gaussian elimination, keeping track of the effect on determinant, multiplying the diagonal entries at the end. This would be $O(n^3)$, the order of Gaussian elimination.

n	2	3	4	5	6	7
n^3	8	27	64	125	216	343
$n!$	2	6	24	120	720	5040

For small values of n the cofactor method wins, but as n grows $n!$ get very big very quickly and the cofactor method becomes impractical.