3

4. Find the eigenvalues for all matrices and eigenvectors for matrix A. Determine which matrices are diagonalizable.

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$

$p(\lambda) = \det(A - \lambda I) = \det\begin{bmatrix} 3-\lambda & 2 \\ 3 & -2-\lambda \end{bmatrix} = (3-\lambda)(-2-\lambda) - 6$

$= \lambda^2 - \lambda - 12$

$\lambda^2 - \lambda - 12 = 0 \Rightarrow (\lambda - 4)(\lambda + 3) = 0 \Rightarrow \lambda = 4$ or $\lambda = -3$

$\lambda = 4$

$(A - 4I)x = 0$

$\begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix} x = 0$

$\left[\begin{array}{cc|c} -1 & 2 & 0 \\ 3 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$x_1 - 2x_2 = 0$

$\vec{x} = \begin{pmatrix} 2x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad \lambda = 4 \quad x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda = -3$

$(A + 3I)x = 0$

$\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} x = 0$

$\left[\begin{array}{cc|c} 6 & 2 & 0 \\ 3 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1/3 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$x_1 + 1/3\, x_2 = 0$

$\vec{x} = \begin{pmatrix} -1/3\, x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -1/3 \\ 1 \end{pmatrix}$ $\quad \lambda = -3 \quad x = \begin{pmatrix} -1/3 \\ 1 \end{pmatrix}$

$$B = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix}$$

$p(\lambda) = \det(B - \lambda I) = 0$

$= \det\begin{bmatrix} -1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 3 & -1-\lambda \end{bmatrix} = 0$

$= (-1-\lambda)\begin{bmatrix} 2-\lambda & 1 \\ 3 & -1-\lambda \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 & 1 \\ 0 & -1-\lambda \end{bmatrix}$

$= (-1-\lambda)[(2-\lambda)(-1-\lambda) - 3] - (-1-\lambda)$

$= (-1-\lambda)[(2-\lambda)(-1-\lambda) - 3 - 1]$

$= (-1-\lambda)(\lambda^2 - \lambda - 6)$

$= (-1-\lambda)(\lambda - 3)(\lambda + 2) = 0$

$\lambda = -1, \lambda = 3,$ or $\lambda = -2$

$$C = \begin{bmatrix} 1 & 6 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$p(\lambda) = \det(C - \lambda I) = 0$

$= \det\begin{bmatrix} 1-\lambda & 6 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{bmatrix} = 0$

$= (1-\lambda) \cdot \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$

$= (1-\lambda)[(2-\lambda)(2-\lambda) - 1] = 0$

$= (1-\lambda)(\lambda^2 - 4\lambda + 3) = 0$

$= (1-\lambda)(\lambda - 3)(\lambda - 1) = 0$

$\lambda = 1, \lambda = 3,$ or $\lambda = 1$

$$D = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$p(\lambda) = \det(D - \lambda I) = 0$

$= \det\begin{bmatrix} 4-\lambda & 0 & -2 \\ 2 & 5-\lambda & 4 \\ 0 & 0 & 5-\lambda \end{bmatrix} = 0$

(Expand across Row 3)

$= (5-\lambda) \cdot \det\begin{bmatrix} 4-\lambda & 0 \\ 2 & 5-\lambda \end{bmatrix} = 0$

$= (5-\lambda)(4-\lambda)(5-\lambda) = 0$

$\lambda = 5, \lambda = 4,$ or $\lambda = 5$