2

3. Assume that $A = [a_1, a_2, a_3, a_4, a_5]$ and $B = [b_1, b_2, b_3, b_4, b_5]$ are row equivalent (i.e. $B = \text{rref}(A)$), where

$$A = \begin{pmatrix} 1 & 2 & -2 & 0 & 7 \\ -2 & -3 & 1 & -1 & -5 \\ -3 & -4 & 0 & -2 & -3 \\ 3 & 6 & -6 & 5 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Find a basis for the column space of A.

Using cols of B to refer back to A.

$$\text{Basis } \text{col}(A) = \left\{ \begin{pmatrix} 1 \\ -2 \\ -3 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ -4 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -2 \\ 5 \end{pmatrix} \right\}$$

(b) Find a basis for the row space of A.

Use Rows of B.

Basis for Row space = $\{(1\ 0\ 4\ 0\ {-3}), (0\ 1\ {-3}\ 0\ 5), (0\ 0\ 0\ 1\ {-4})\}$

(c) Find a basis for the null space of A.

Solve $Ax=0$, use B.

$$\left[\begin{array}{ccccc|c} 1 & 0 & 4 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 4x_3 - 3x_5 = 0$
$x_2 - 3x_3 + 5x_5 = 0$
$x_4 - 4x_5 = 0$

$$\vec{x} = \begin{pmatrix} -4x_3 + 3x_5 \\ 3x_3 - 5x_5 \\ x_3 \\ 4x_5 \\ x_5 \end{pmatrix} = x_3 \begin{pmatrix} -4 \\ 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 3 \\ -5 \\ 0 \\ 4 \\ 1 \end{pmatrix}$$

$$\text{Basis Null Space} = \left\{ \begin{pmatrix} -4 \\ 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \\ 0 \\ 4 \\ 1 \end{pmatrix} \right\}$$

(d) Find the dim(Nul(A)) and dim(Col(A)).

dim(Nul(A)) = # of Basis vectors = 2
dim(Col(A)) = # of Basis vectors = 3