

# ANSWERS

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MATH 215

Practice Exam 3 (4.1, 4.2, 4.3, 4.4, 4.5, 5.1, 5.2, 5.3)

1. Let  $S = \{(a-b, b-a) \mid a \text{ and } b \text{ are real numbers}\}$  be a subset of  $\mathbb{R}^2$ . Show that  $S$  is a subspace of  $\mathbb{R}^2$ . Find the dimension of  $S$ .

Method 1

$$\begin{pmatrix} a-b \\ b-a \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$= \text{span} \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$$

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} = -1 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \text{span} \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\} \checkmark$$

2. Let  $H = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : 3a - 2b = 5c \right\}$ . Show  $H$  is a subspace of  $\mathbb{R}^3$  and find the dimension of  $H$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ such that } 3a - 2b = 5c$$

$$3a - 2b - 5c = 0$$

$$\begin{bmatrix} 3 & -2 & -5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

Find the nullspace of  $\begin{bmatrix} 3 & -2 & -5 \end{bmatrix}$

Method 2

①  $\vec{0} \in S$ , set  $a=0$  &  $b=0$  ✓

②  $u, v \in S \Rightarrow u = \begin{pmatrix} u_1 - u_2 \\ u_2 - u_1 \end{pmatrix}, v = \begin{pmatrix} v_1 - v_2 \\ v_2 - v_1 \end{pmatrix}$

$$u+v = \begin{pmatrix} (u_1+v_1) - (u_2+v_2) \\ (u_2+v_2) - (u_1+v_1) \end{pmatrix} \in S \checkmark$$

③  $\alpha \in \mathbb{R} \Rightarrow \alpha u \in S \Rightarrow u = \begin{pmatrix} u_1 - u_2 \\ u_2 - u_1 \end{pmatrix}$

$$\alpha u = \begin{pmatrix} \alpha u_1 - \alpha u_2 \\ \alpha u_2 - \alpha u_1 \end{pmatrix} \checkmark$$

$$\rightarrow \text{ref} \Rightarrow \left[ 1 \quad -2/3 \quad -5/3 \mid 0 \right]$$

$$a - 2/3 b - 5/3 c = 0 \quad \text{free vars: } b, c$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2/3 b + 5/3 c \\ b \\ c \end{pmatrix} = b \cdot \begin{pmatrix} 2/3 \\ 1 \\ 0 \end{pmatrix} + c \cdot \begin{pmatrix} 5/3 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{Null space} = \text{span} \left\{ \begin{pmatrix} 2/3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 5/3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

This is a basis.

Null space is a subspace ✓

Basis two vectors  $\Rightarrow \dim(H) = 2$  ✓