Chapter 21: Saving Models

Section 21.5 Present Value and Inflation

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- Present Value
  - The present value of an amount to be paid or received at a specific time in the future is what the future payment would be worth today, as determined from a given interest rate and compounding period.
  - The present value P of an amount A to be paid t years in the future, earning a nominal annual interest rate r compounded m times per year—that is, after $n = mt$ compounding periods at rate of $i = r/m$ per compounding period—is:

$$P = \frac{A}{(1+i)^n} = \frac{A}{\left(1+\frac{r}{m}\right)^{mt}}$$

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- Inflation
  - Inflation is a rise in prices from a set base year.
  - The annual rate of inflation *a* (given as %) (convert to decimal for use in formula), is the additional proportionate cost of goods one year later. Goods that cost \$1 in the base year will then cost \$(1 + *a*).

    Example: If the inflation rate is *a = 25%*, then what cost \$1 now would cost \$1.25 this time next year.

    - Example: Suppose that there was a constant 3% annual inflation from mid-2009 until mid-2013. What would be the projected price in mid-2013 of an item that costs \$100 in mid-2009?
      - Solution: Using the compound interest formula with $P = \$100$, $r = 3\% = .03$, $m = 1$, and $t = 4$:

$$A = P\left(1 + r\right)^t = \$100\left(1 + .03\right)^4 = \$112.55$$

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Exponential Decay

- Exponential decay is geometric growth with a negative rate of growth.
- Present Value of a Dollar a Year from Now with Inflation Rate a.

$$\frac{\$1}{1+a} = \$1 - \frac{\$a}{1+a}$$

The quantity i=–a/(1+a) behaves like a negative interest rate so we can use the compound interest formula to find the present value of P dollars t years from now.

- Example: Suppose a 25% annual inflation rate from mid-2009 through mid-2013. What will be the value of a dollar in mid-2013 in constant mid-2009 dollars?

Answer: a=0.25, so i=-a/(1+a)=-0.25/(1+.25)=-0.25/1.25=-0.20 and

$$P(1+i)^t = \$1(1-0.20)^4 = (0.80)^4 = \$0.41$$

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- Depreciation Example

Suppose you bought a car at the beginning of 2009 for $12,000 and its value in current dollars depreciates steadily at a rate of 15% per year. What will be its value at the beginning of 2012 in current dollars?

Answer: Using the compound interest formula, $P = \$12{,}000$, $i = -0.15$, and $n = 3$. The projected price is $A = P\,(1 + r)^n = \$12{,}000\,(1 - 0.15)^3 = \$7369.50$

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Consumer Price Index

- The official measure of inflation is the Consumer Price Index (CPI), prepared by the Bureau of Labor Statistics.
  - This index represents all urban consumers (CPI-U) and covers about 80% of the U.S. population.
  - This is the index of inflation that is referred to on television news broadcasts, in newspapers, and magazine articles.
  - Each month, the Bureau of Labor Statistics determines the average cost of a "market basket" of goods, including food, housing, transportation, clothing, and other items.
  - The base period used to construct the CPI-U is from 1982–1984 and is set to 100.

$$\frac{\text{CPI for other year}}{100} = \frac{\text{cost of market basket in other year}}{\text{cost of market basket in base year}}$$

  - From this proportion calculation, you can also compute the cost of an item in dollars for one year to what it would cost in dollars in a different year.

$$\frac{\text{cost in year A}}{\text{cost in year B}} = \frac{\text{CPI for year A}}{\text{CPI for year B}}$$

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$$\frac{\text{Cost in year A}}{\text{Cost in year B}} = \frac{\text{CPI for year A}}{\text{CPI for year B}}$$

Example: The average cost of a Madison house in 1976 is $38,323. What is the average cost of a home in Madison in 2003?

$$\frac{\text{Cost in 2003}}{\text{Cost in 1976}} = \frac{\text{CPI for 2003}}{\text{CPI for 1976}} \longrightarrow \frac{\text{Cost in 2003}}{\$38,323} = \frac{184}{56.9}$$

Thus Cost in 2003 = $38,323 x 184/56.9 = $123,926.75

Old Exam Question

Betty bought a house in 1987 for $99,000 and sold it in 2001. If the 1987 CPI is 113.6 and the 2001 CPI is 177.7, how much would the house be worth in 2001 dollars?

$$\frac{\text{Cost in 2001}}{\text{Cost in 1987}} = \frac{\text{CPI for 2001}}{\text{CPI for 1987}}$$

$$\frac{\text{Cost in 2001}}{99000} = \frac{177.7}{113.6}$$

$$\text{Cost in 2001} = \$99000 \times \frac{177.7}{113.6} = \$154861.79$$