Chapter 2: Urban Services

For All Practical Purposes

Mathematical Literacy in Today's World, 9th ed.

Section 2.4 Minimum-Cost Spanning Trees

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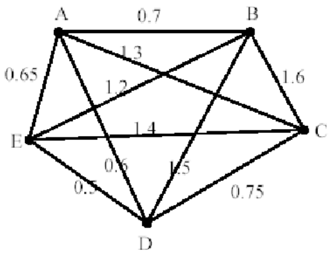
Pictaphone Service
between cities



The **possible links** **that might be included in the network, with each edge showing the cost in mi dollars to create that particular link**

Pictaphone Service

1. A **direct** communication link is not necessary for sending a Pictaphone message between two cities

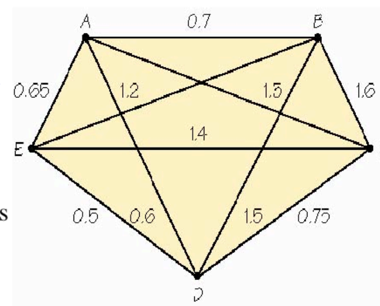
2. The cost of relaying a message, compared with the direct communication link cost, can be neglected.

Goal: Provide service between any pair of cities in a way that minimizes the total cost of the links.

- <u>Minimum-Cost Spanning Trees</u>
  - Another graph theory optimization problem that links all the vertices together, in order of increasing costs, to form a "tree."
  - The cost of the tree is the sum of the weights on the edges.

Example: What is the cost to construct a Pictaphone service (telephone service with video image of the callers) among five cities?

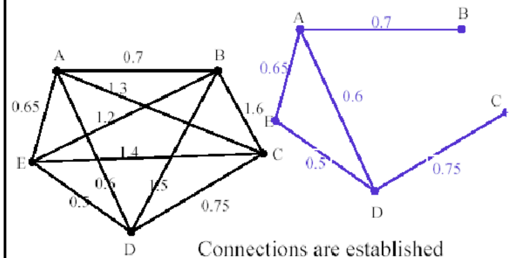
- The diagram shows the cost to build the connection from each vertex to all other vertices (connected graph).
- Cities are linked in order of increasing costs to make the connection.
- The cost of redirecting the signal may be small compared to adding another link.



Costs (in millions of dollars) of installing Pictaphone service among five cities

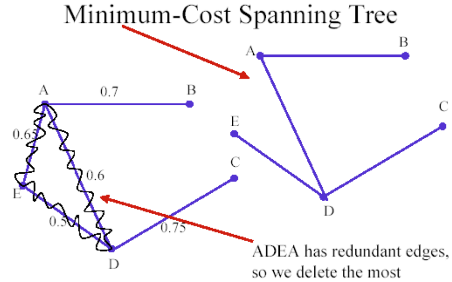
3

Minimum-Cost Spanning Trees



Connections are established among five cities A, B, C, D, E using the minimum-cost spanning

Minimum-Cost Spanning Tree



ADEA has redundant edges, so we delete the most expansive edge **AE**.

- Kruskal's Algorithm — Developed by Joseph Kruskal (AT&T research).
  - Goal of minimum-cost spanning tree: Create a tree that links all the vertices together and achieves the lowest cost to create.
  - Add links in order of cheapest cost according to the rules:
    - No circuit is created (no loops).
      - If a circuit (or loop) is created by adding the next largest link, eliminate this largest (most expensive link)—it is not needed.
    - Every vertex must be included in the final tree.

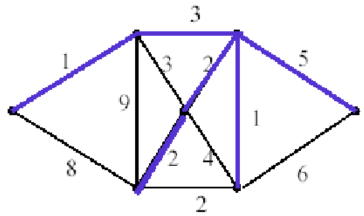
Minimum-Cost Spanning Trees: Summary

Approach: add the links in order of cheapest cost so that no circuits form and so that every vertex belongs to some link added.

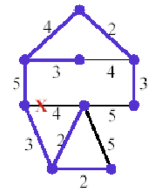
This approach is called Kruskal's algorithm.

4

Use Kruskal's algorithm for minimum-cost spanning trees on the graph below. The cost of the three is:



Use Kruskal's algorithm for minimum-cost spanning trees on the graph below. What is the cost of the tree found?



X A. 24

__B. 26

__C. 31

__D. 42

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5

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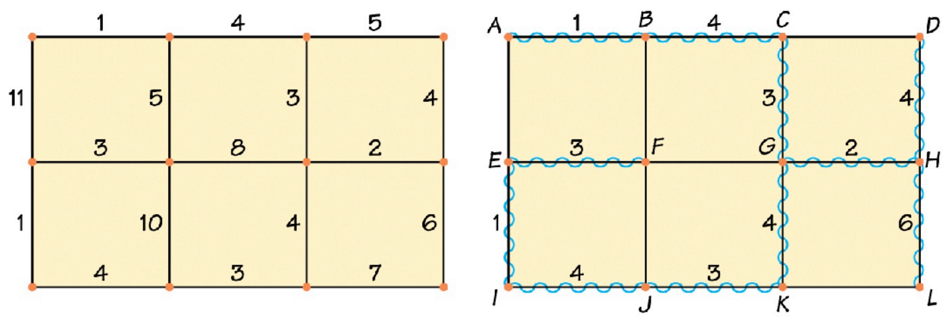

(a) (b)

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