MATH 215
Practice

1. Solve the given systems of equations. Show **ALL** row operations that you used.

$$\begin{aligned} 4x_1 - 3x_2 &= 10 \\ 8x_1 - x_2 &= 10 \end{aligned}$$

$$\left[\begin{array}{cc|c} 4 & -3 & 10 \\ 8 & -1 & 10 \end{array}\right] \overset{\text{rref}}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array}\right]$$

$$\left.\begin{aligned} x_1 &= 1 \\ x_2 &= -2 \end{aligned}\right\} \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\boxed{\text{ANS: } \vec{x} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}}$$

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ 3x_1 + x_2 + 4x_3 &= 0 \\ 2x_1 + 2x_2 + 3x_3 &= 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 3 & 1 & 4 & 0 \\ 2 & 2 & 3 & 2 \end{array}\right] \overset{\text{rref}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\left.\begin{aligned} x_1 &= -3 \\ x_2 &= 1 \\ x_3 &= 2 \end{aligned}\right\} \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

$$\boxed{\text{ANS: } \vec{x} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}}$$

$$\begin{aligned} x_1 + 4x_2 - 2x_3 &= 4 \\ 2x_1 + 7x_2 - x_3 &= -2 \\ 2x_1 + 9x_2 - 7x_3 &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -2 & 4 \\ 2 & 7 & -1 & -2 \\ 2 & 9 & -7 & 1 \end{array}\right] \overset{\text{rref}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\boxed{\text{ANS: No solution}}$$

$$\begin{aligned} x_1 - 3x_2 + 2x_3 - x_4 &= 8 \\ 3x_1 - 7x_2 \qquad\quad + x_4 &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & -3 & 2 & -1 & 8 \\ 3 & -7 & 0 & 1 & 0 \end{array}\right] \overset{\text{rref}}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & -7 & 5 & -28 \\ 0 & 1 & -3 & 2 & -12 \end{array}\right]$$

$$\begin{aligned} x_1 - 7x_3 + 5x_4 &= -28 \\ x_2 - 3x_3 + 2x_4 &= -12 \end{aligned}$$

$$\begin{aligned} x_1 &= -28 + 7x_3 - 5x_4 \\ x_2 &= -12 + 3x_3 - 2x_4 \\ x_3 &= x_3 \\ x_4 &= x_4 \end{aligned}$$

$$\boxed{\text{ANS: } \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -28 \\ -12 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 7 \\ 3 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -5 \\ -2 \\ 0 \\ 1 \end{pmatrix}}$$

$$A = \begin{pmatrix} 1 & 2 & 4 \\ -2 & -3 & -5 \\ 2 & 1 & -1 \end{pmatrix}$$

2. Are the columns of A linearly independent or linearly dependent?

$$\begin{pmatrix} 1 & 2 & 4 \\ -2 & -3 & -5 \\ 2 & 1 & -1 \end{pmatrix} \overset{rref}{\sim} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

Columns of A are Linearly dependent

$$\begin{pmatrix} 4 \\ -5 \\ -1 \end{pmatrix} = -2\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

3. Do the columns of A span $\mathbf{R}^3$?

$$\begin{pmatrix} 1 & 2 & 4 \\ -2 & -3 & -5 \\ 2 & 1 & -1 \end{pmatrix} \overset{rref}{\sim} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

No. Must have a pivot in all rows. Last row of zeros has no pivot. If solving $A\vec{x} = \vec{b}$ for any vector $\vec{b}$, will cause problem in the last row, i.e. restriction on vector $\vec{b}$.

4. Let $b = \begin{pmatrix} -5 \\ 6 \\ 2 \end{pmatrix}$ Does $Ax = b$ have a solution? If so find x, if not state why.

$$\left(\begin{array}{ccc|c} 1 & 2 & 4 & -5 \\ -2 & -3 & -5 & 6 \\ 2 & 1 & -1 & 2 \end{array}\right) \overset{rref}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Yes. Last row all zeros is O.K. $\vec{b} = \begin{pmatrix} -5 \\ 6 \\ 2 \end{pmatrix}$ is a specific $\vec{b}$ and is in the span of the columns of A.

$$\begin{cases} x_1 - 2x_3 = 3 \\ x_2 + 3x_3 = -4 \end{cases}$$

$$\boxed{\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}}$$

$x_1 = 3 + 2x_3$

$x_2 = -4 - 3x_3$

$x_3 = x_3$

5. Is the vector $b = \begin{pmatrix} 3 \\ 5 \\ 8 \end{pmatrix}$ in the span $\left\{ \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ 5 \end{pmatrix} \right\}$?

$$\left(\begin{array}{ccc|c} 0 & 1 & -3 & 3 \\ 2 & 4 & -1 & 5 \\ 4 & -2 & 5 & 8 \end{array}\right) \overset{\text{rref}}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 147/46 \\ 0 & 1 & 0 & -15/23 \\ 0 & 0 & 1 & -28/23 \end{array}\right)$$

Yes $\vec{b}$ is in the span.

6. Determine if the following set of vectors are linearly independent or linearly dependent.

$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right\}$ solve $c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \overset{\text{rref}}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Yes. Vectors are linearly independent, since the only solution is $c_1 = c_2 = c_3 = 0$. Last row of zeros does not imply anything when discussing linear independence.