Chapter 1: Urban Services

Section 1.2 Finding Euler Circuits

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Finding an Euler Circuit

Definitions:

Valence - The valance of a vertex in a graph is the number of edges meeting at the vertex.

Connected – A graph is said to be connected if for every pair of its vertices there is at least one path connecting the two vertices.
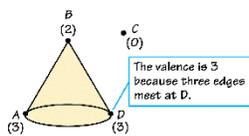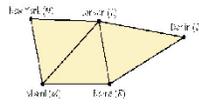
Connected   Not connected
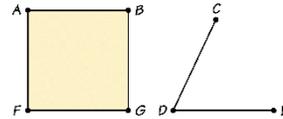
Not Connected

1. If a graph G is **connected** and has all valences **even**, then G has an Euler circuit.

2. Conversely, if G has Euler circuit, then G must be connected and all its valences must be even numbers

Odd number of valences, therefore no Euler circuit can exist.

All vertices have even valence. This has an Euler Circuit.
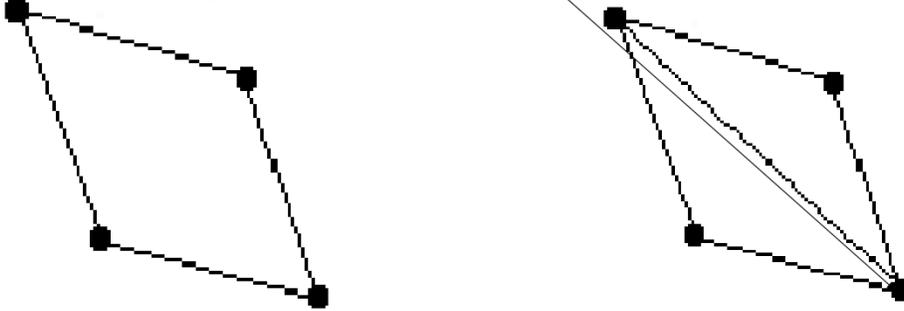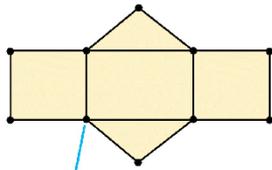
(a)

All the valences are even, so the graph has an Euler circuit.

(b)

Start here
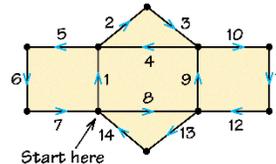
- **Two Ways to Find an Euler Circuit**
  - ❑ Trial and error

    Keep trying to create different paths to find one that starts and ends at the same point and does not retrace steps.

  - ❑ Mathematical approach (better method)

    An Euler circuit exists if the following statements are true:
    - ➢ All points (vertices) have even valence.
    - ➢ The graph is connected
- Never use an edge that is the only link between two parts of the graph that still need to be covered.

The Euler circuit on the right was found by trial and error.

The network for the **Könisgberg Bridge Problem.**
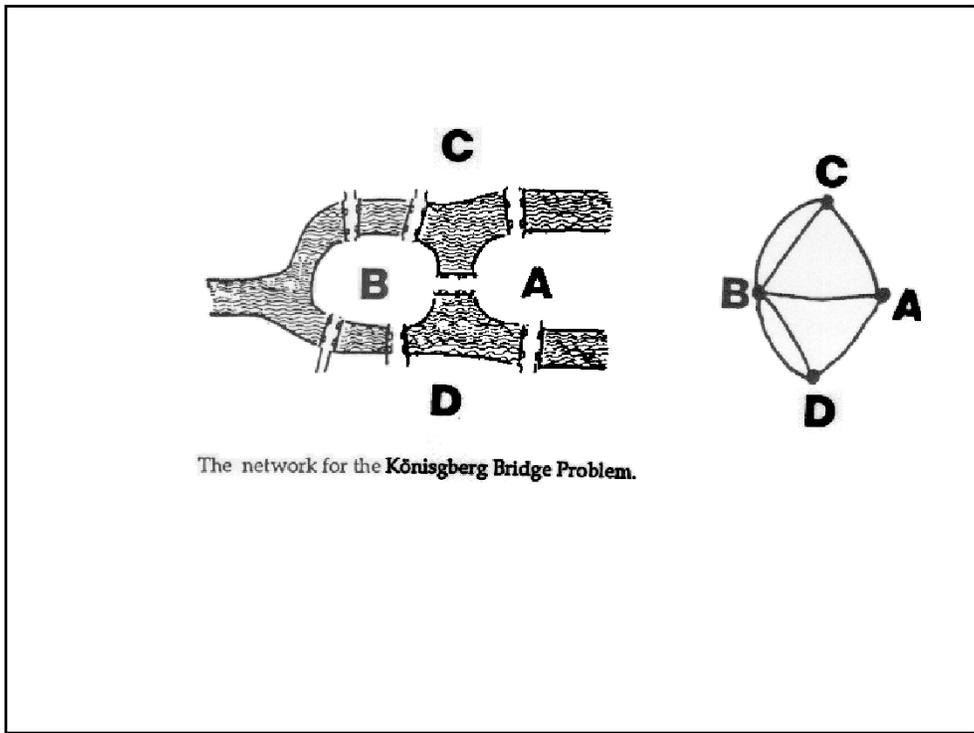
Understand why the Konisberg bridge cannot have an Euler Circuit? Hint: Valence.

■ **Is there an Euler Circuit?**

❏ Does it have even valence?

❏ Is the graph connected?

Euler circuit exists if both "yes."

■ Create (Find) an Euler Circuit

❏ Pick a point to start (if none has been given to you).

❏ Number the edges in order of travel, showing the direction with arrows.

❏ Cover every edge only once, and end at the same vertex where you started.
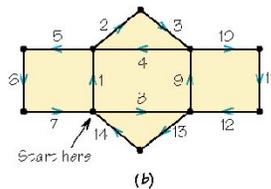
A Guidance for Finding Euler Circuits
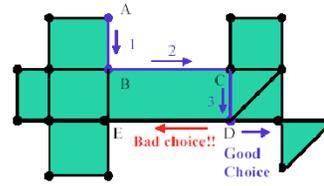
Never use an edge that is the only one link between two parts of the graph that still need to be covered.

Finding Euler Circuits

• **Trial and Error**. Draw your graph in ink and the circuit in pencil so you can erase.

• **Big Graph**. Make your graph big and clear so you won't get confused.

• A graph may have morn than one Euler circuit.

(b)
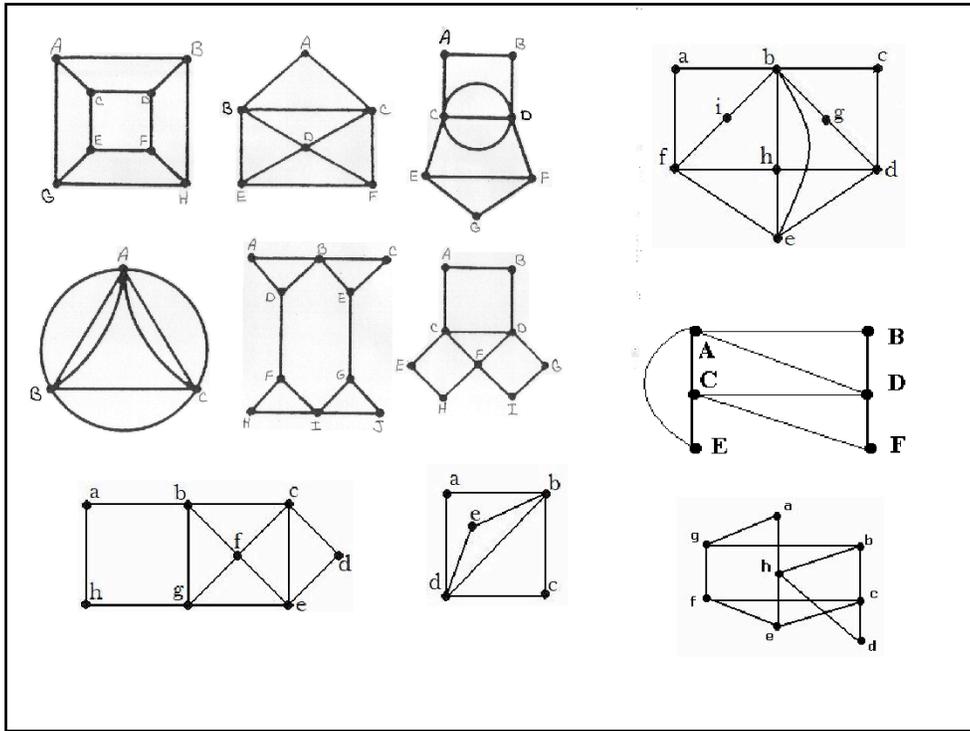
Try finding Euler circuits for the problems listed above. Practice, practice!!!

Answers