Chapter 1: Urban Services

Section 1.1 Euler Circuits

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Chapter 1: Urban Services

- <u>Management Science</u>
  - Uses mathematical methods to help find optimal solutions to management problems. Often called Operations Research.
- <u>Optimal Solutions</u> — The best (most favorable) solution
  - Government, business, and individuals all seek optimal results.
    - Optimization problems:
      - Finish a job quickly
      - Maximize profits
      - Minimize costs
    - Urban Services to optimize:
      - Checking parking meters
      - Delivering mail
      - Removing snow

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Chapter 1:   Street Networks

Goals: 1.) Cover all sidewalks that have parking meters without retracing any more steps than are necessary.
2.) Route should start and end at the same point.

Definitions:

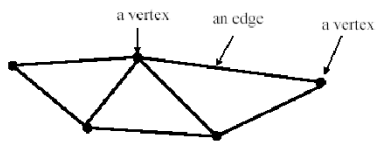**Graph** – Finite set of dots and connecting links.

Vertices – The dots (a single dot is a **vertex**).
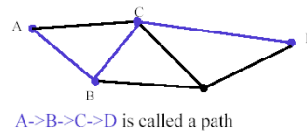
Edges – The links.

Path – Connected sequence of edges that starts at a vertex and ends at a vertex.

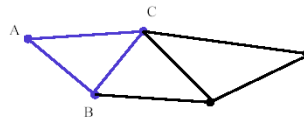**Circuit** – A path that starts and ends at the same vertex.

A Graph

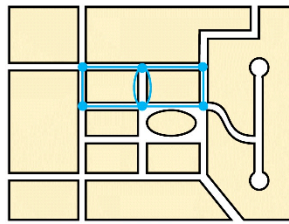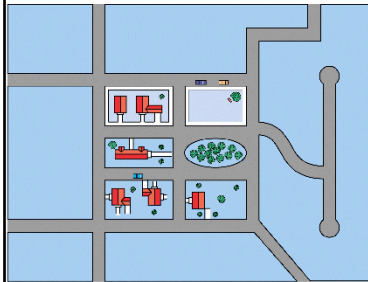a vertex   an edge   a vertex
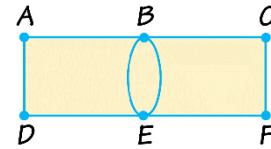
A Path

A->B->C->D is called a path

A Circuit

The path A->B->C->A is called a circuit.
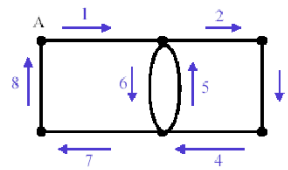
The case of parking control

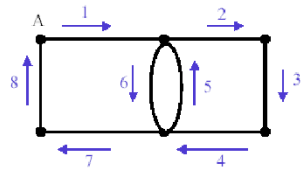Edges show which sidewalks have parking meters.

(a)

(b)

No edge is covered morn than once!

Assume: we start and end at A.

Question: can we find an efficient route starting and ending at A?
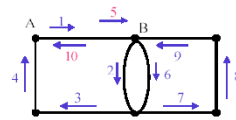
An Euler Circuit

Circuits that cover <u>every edge only</u> <u>once</u> are called Euler circuits.

Not an Euler circuit

Note: edge AB is covered morn than once!

Thus this path is NOT an Euler circuit.

Leonhard Euler (1707-1783)

The Königsberg Bridge Problem & Topology

Topology originated with the solution in 1736 of a famous problem — *the Königsberg Bridge Problem.*

Königsberg[1] is a city on the Preger River that contains two islands and is joined by seven bridges. The river flows around the two islands of the town. The bridges run from the banks of
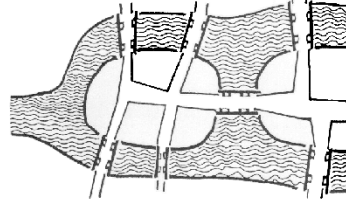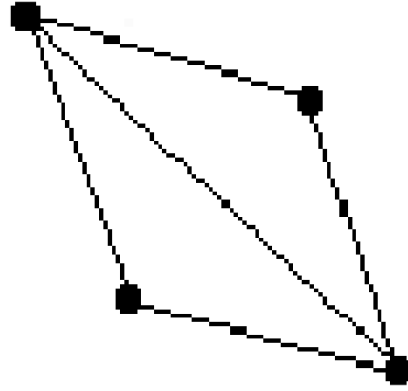

Diagram of the Königsberg Bridge Problem.

the river to the two islands in the river with a bridge connecting the islands. It became a town tradition to take a Sunday walk, and try to cross each of the seven bridges only once. No one had solved the problem until it came to the attention of the Swiss mathematician Leonhard Euler (1707-1783). At that time, Euler was serving the Russian empress Catherine the Great in St. Petersburg. In the process of solving this problem, Euler invented the branch of mathematics known as topology. He solved the *Königsberg Bridge Problem* by using an area of topology today called networks. Using networks, he proved that the problem of crossing each bridge of Königsberg only once was not possible.

Some graphs have no Euler circuits
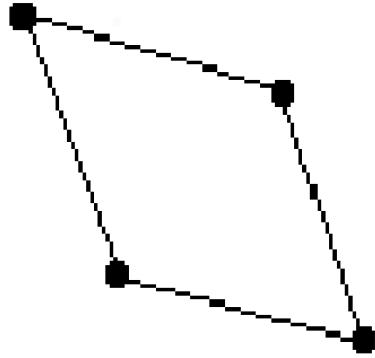
Euler Circuit   Not an Euler Circuit