For All Practical Purposes

Mathematical Literacy in Today's World, 9th ed.

Chapter 9: Social Choice: The Impossible Dream

Section 9.2 Majority Rule and Condorect's Method

James Baglama
Department of Mathematics
University of Rhode Island

Voting and Social Choice

- Social Choice Theory
  - Social choice deals with how groups can best arrive at decisions.
  - The problem with social choice is finding good procedures that will turn individual preferences for different candidates into a single choice by the whole group.

Example: Selecting a winner of an election using a good procedure that will result in an outcome that "reflects the will of the people"

Preference List Ballot

- ❑ A preference list ballot consists of a rank ordering of candidates showing the preferences of one of the individuals who is voting.
- ❑ A vertical list is used with the most preferred candidate on top and the least preferred on the bottom.

Throughout the chapter, we assume the number of voters is odd (to help simplify and focus on the theory). Furthermore, in the real world, the number of voters is often so large that ties seldom occur.

2

Majority Rule

- <u>Majority Rule</u>
  - Majority rule for elections with only two candidates (and an odd number of voters) is a voting system in which the candidate preferred by more than half the voters is the winner.
- <u>Three Desirable Properties of Majority Rule</u>
  - All voters are treated equally.
  - Both candidates are treated equally.
  - It is monotone.

<u>Monotone</u> means that if a new election were held and a single voter were to change his or her ballot from voting for the losing candidate to voting for the winning candidate (and everyone else voted the same), the outcome would be the same.

<u>May's Theorem</u> – Among all two-candidate voting systems that never result in a tie, majority rule is the only one that treats all voters and both candidates equally and is monotone.

Condorcet's Method (3 or more candidate's)

- Condorcet's Method
  - This method requires that each candidate go head-to-head with each of the other candidates.

- For the two candidates in each contest, record who would win (using majority rule) from each ballot cast. To satisfy Condorcet, the winning candidate must defeat every other candidate one-on-one.
- The Marquis de Condorcet (1743 – 1794) was the first to realize the voting paradox: If A is better than B, and B is better than C, then A must be better than C. **Sometimes C is better than A—not logical!**

Condorcet's Voting Paradox – With three or more candidates, there are elections in which Condorcet's method yields no winners.

A beats B, 2 out of 3; and
B beats C, 2 out of 3; and
C beats A, 2 out of 3 — No winner!

Rank	Number of Voters (3)		
First	A	B	C
Second	B	C	A
Third	C	A	B

4

EXAMPLE 1 ■ Condorcet's Method

Suppose we have four candidates (*GB, AG, RN,* and *PB,* with these initials chosen for a soon-to-be-revealed reason) and the following sequence of preference list ballots, where the heading of "6" indicates that 6 of the 15 voters hold the ballot with *GB* over *AG* over *PB* over *RN,* the heading of "5" indicates that 5 of the 15 voters hold the ballot with *AG* over *RN* over *GB* over *PB,* and so on.

	Number of Voters (15)			
Rank	6	5	3	1
First	*GB*	*AG*	*RN*	*PB*
Second	*AG*	*RN*	*AG*	*GB*
Third	*PB*	*GB*	*GB*	*AG*
Fourth	*RN*	*PB*	*PB*	*RN*

We claim that *AG* is the winner in this election if we use Condorcet's method. Let's check the one-on-one scores for each possible pair of opponents:

AG versus *GB: AG* is over *GB* on 5 + 3 = 8 of the ballots, while the reverse is true on 6 + 1 = 7 of the ballots. Thus, *AG* defeats *GB* by a score of 8 to 7.

AG versus *RN: AG* is over *RN* on 6 + 5 + 1 = 12 of the ballots, while the reverse is true on 3 of the ballots. Thus, *AG* defeats *RN* by a score of 12 to 3.

AG versus *PB: AG* is over *PB* on 6 + 5 + 3 = 14 of the ballots, while the reverse is true on 1 of the ballots. Thus, *AG* defeats *PB* by a score of 14 to 1.

This shows that *AG* is the winner using Condorcet's method.