Math108 — Spring 2009 — HW #3 — Solutions

1. (pg. 112, #4) Malia buys the chocolate-strawberry-vanilla cake shown in (i) for \$11.20. Malia values strawberry twice as much as chocolate and values chocolate twice as much as vanilla.

   (a) What is the value of the chocolate part of the cake to Malia?

$$\begin{aligned} S + C + V &= 11.20 \\ S &= 4V \\ C &= 2V \\ 4V + 2V + V &= 11.20 \\ 7V &= 11.20 \\ V &= 1.60 \\ C &= 3.20 \\ S &= 6.40 \end{aligned}$$

   Chocolate: \$3.20

   (b) What is the value of the strawberry part of the cake to Malia?
   Strawberry: \$6.40

   (c) What is the value of the vanilla part of the cake to Malia?
   Vanilla: \$1.60

   (d) If the cake is cut into six equal sized pieces shown in (ii), find the value to Malia of each of the six pieces.

$$\begin{array}{lcl} Piece1 = & \frac{1}{2}V & = \$0.80 \\ Piece2 = & \frac{1}{4}V + \frac{1}{4}S & = \$2.00 \\ Piece3 = & \frac{1}{2}S & = \$3.20 \\ Piece4 = & \frac{1}{4}S + \frac{1}{4}C & = \$2.40 \\ Piece5 = & \frac{1}{2}C & = \$1.60 \\ Piece6 = & \frac{1}{4}C + \frac{1}{4}V & = \$1.20 \end{array}$$

2. (pg. 112, # 6) Three players (Alex, Betty, and Cindy) must divide a cake among themselves. Suppose the cake is divided into three slices (s_1, s_2, and s_3). The following table shows the percentages of the value of the entire cake that each slice represents to each player.

	s_1	s_2	s_3
Alex	30%	40%	30%
Betty	35%	25%	40%
Cindy	$33\frac{1}{3}$%	50%	$16\frac{1}{3}$%

   Recall: A piece is considered a fair share if its value is at least $33\frac{1}{3}\%$.

   (a) Indicate which of the three slices are fair shares to Alex.
   s_2

   (b) Indicate which of the three slices are fair shares to Betty.
   s_1 or s_3

(c) Indicate which of the three slices are fair shares to Cindy.

s_1 or s_2

(d) Describe a fair division of the cake.

In any fair division, Alex must have s_2, so give s_2 to Alex. Then Cindy must get s_1. And lastly Betty must get s_3.

3. (pg. 114, # 14) Raul and Karli are planning to divide a chocolate-strawberry mousse cake in the figure using the divider-chooser method. Raul values chocolate three times as much as he vlues strawberry. Karli values chocolate twice as much as she values strawberry.

(a) If Raul is the divider, which of the cuts shown in figures (i) through (v) are consistent with Raul's value system.

Recall: A division is consistent with Raul's value system if he values all of its pieces equally.

Cut (i) is consistent as the two pieces are identical.

Cut (ii) is not consistent because Raul values the top half three times as much as the bottom half.

For cuts (iii)-(v), let's say Raul values the chocolate portion at 3 and values the strawberry portion at 1.

Cut (iii) is consistent with Raul's value system, as he values each piece at 2.

Cut (iv) is not consistent with Raul's value system. He values the left piece at $\frac{60}{180}3 + \frac{72}{180}1 = 1 + \frac{2}{5} = \frac{7}{5}$. He then values the right piece at $4 - \frac{7}{5} = \frac{13}{5}$, showing he values the two pieces differently.

Cut (v) is consistent with Raul's value system. He values the left piece at $\frac{96}{180}3 + \frac{72}{180}1 = \frac{8}{5} + \frac{3}{5} = 2$, so he values the right piece at $4 - 2 = 2$ as well.

(b) For each of the cuts consistent with Raul's value system, indicate which of the pieces is Karli's best choice.

From cut (i), Karli will choose either piece, as she values the two pieces equally.

For cuts (iii) and (v), let's say Karli values the chocolate portion at 2 and the strawberry portion at 1.

From cut (iii), Karli values the pure chocolate piece at $\frac{2}{3}2 = \frac{4}{3}$ and the other piece at $3 - \frac{4}{3} = \frac{5}{3}$, so she would pick the chocolate-strawberry mixed piece.

From cut (v), Karli values the left piece at $\frac{96}{180}2 + \frac{72}{180}1 = \frac{16}{15} + \frac{2}{5} = \frac{22}{15}$, and the right piece at $3 - \frac{22}{15} = \frac{23}{15}$, so she would pick the right piece.

4. (pg. 120, # 54) Andre, Bea, and Chad are dividing an estate consisting of a house, a small farm, ad a painting, using the method of sealed bids. Their bids on each of the items are given in the following table.

	Andre	Bea	Chad
House	\$150,000	\$146,000	\$175,000
Farm	\$430,000	\$425,000	\$428,000
Painting	\$50,000	\$59,000	\$57,000

First, we'll solve it the way we did in class.

(a) Describe the first sttlement of this fair division and compute the surplus.

House:
Andre **gets** $\frac{1}{3}\$150,000 = \$50,000$ **from** the kitty.
Bea **gets** $\frac{1}{3}\$146,000 = \$48,667$ **from** the kitty.
Chad gets the house and **pays** $\frac{2}{3}\$175,000 = \$116,667$ **to** the kitty.

Farm:
Andre gets the farm and **pays** $\frac{2}{3}\$430,000 = \$286,667$ **to** the kitty.
Bea **gets** $\frac{1}{3}\$425,000 = \$141,667$ **from** the kitty.
Chad **gets** $\frac{1}{3}\$428,000 = \$142,667$ **from** the kitty.

Painting:
Andre **gets** $\frac{1}{3}\$50,000 = \$16,667$ **from** kitty.
Bea gets the painting and **pays** $\frac{2}{3}\$59,000 = \$39,333$ **to** the kitty.
Chad **gets** $\frac{1}{3}\$57,000 = \$19,000$ **from** the kitty.

The kitty now contains $-\$50,000-\$48,667+\$116,667+\$286,667-\$141,667-\$142,667-\$16,667+\$39,333-\$19,000 = \$24,000$.

(b) Desribe the final settlement of this fair-division problem.

Each person gets one-third of the surplus, or \$8,000. Then we have that:

Andre gets the farm and pays \$212,000.
Bea gets the painting and \$159,000.
Chad gets the house and \$53,000.

Now, we will solve the problem using the method given in the textbook.

(a) Describe the first settlement of this fair division and compute the surplus.

The house goes to Chad. The farm goes to Andre. The painting goes to Bea.

Andre's fair-dollar share is $(\$150,000 + \$430,000 + \$50,000)/3 = \$210,000$, and he received the farm which he values at \$430,000, so he owes $\$430,000 - \$210,000 = \$220,000$.

Bea's fair-dollar share is $(\$146,000 + \$425,000 + \$59,000)/3 = \$210,000$, and she received the painting which she values at \$59,000, so she is owed $\$210,000 - \$59,000 = \$151,000$.

Chad's fair-dollar share is $(\$175,000 + \$428,000 + \$57,000)/3 = \$220,000$, and he received the house which he values at \$175,000, so he is owed $\$220,000 - \$175,000 = \$45,000$.

The surplus then is $\$220,000 - \$151,000 - \$45,000 = \$24,000$.

(b) Desribe the final settlement of this fair-division problem.

Each person receives one-third of the surplus, or \$8,000. So:

Andre gets the farm and pays $\$220,000 - \$8,000 = \$212,000$.

Bea gets the painting and receives $\$151,000 + \$8,000 = \$159,000$.

Chad gets the house and receives $\$45,000 + \$8,000 = \$53,000$.

5. (pg. 122, # 64) Four players (A, B, C, and D) are dividing the array of 15 items shown in the following figure using the method of markers. The players' bids are indicated in the figure.

1	2	3		4		5		6		7		8		9		10		11		12		13	14		15
			c_1		b_1		c_2		a_1		a_2		b_2		d_1		d_2		a_3 d_3		c_3			b_3	

(a) Describe the allocation of items to each player.

We start at the left, and move forward to the first "1" marker, which is c_1. **C gets 1 through 3.** Ignoring all "c" markers, we continue to the first "2" marker, which is a_2, and give A everything between this and the previous "a" marker. **A gets 7.** Ignoring all "c" and "a" markers, we continuous to the first "3" marker, which is d_3, and give D everything between this and the previous "d" marker. **D gets 11.** Lastly, we give B everything to the right of marker b_3. **B gets 15.**

In summary, A get 7, B gets 15, C gets 1, 2, and 3, and D gets 11.

(b) Which items are left over?

4, 5, 6, 8, 9, 10, 12, 13, 14