MTH 307 Worksheet I Spring 2008

Use the following sets for exercises 1-3.

\[ A = \{-3, -2, -1, 0, 1, 2, 3\} \]
\[ B = \{x \in \mathbb{Z} \mid x^2 < 12\} \]
\[ C = \{x \in \mathbb{Z} \mid 2x - 3 \leq 7\} \]
\[ D = \{y \in \mathbb{Z} \mid |y| \leq 5\} \]
\[ S = \{x \in \mathbb{R} \mid 2x - 3 \leq 7\} \]
\[ T = \{y \in \mathbb{R} \mid |y| \leq 5\} \]

(1) (a) Write the elements of the set \( B \) using the roster method.

(b) Is the set \( A \) equal to the set \( B \)? Explain.

(c) Is \( A \) a subset of \( B \)? Explain.

(d) Is \( A \) a proper subset of \( B \)? Explain.

(e) Is \( B \) a subset of \( A \)? Explain.

(2) (a) Write the elements of \( C \) and \( D \) using the roster method.

(b) Is the set \( C \) equal to the set \( D \)? Explain.

(c) Is \( C \) a subset of \( D \)? Explain.

(d) Is \( D \) a subset of \( C \)? Explain.

(e) Is \( D \) a proper subset of \( C \)? Explain.
(3) (a) Is it possible to write the elements of $S$ and $T$ using the roster method? Explain.

(b) Is the set $S$ equal to the set $T$? Explain.

(c) Is $S$ a subset of $T$? Explain.

(d) Is $T$ a subset of $S$? Explain.

(e) Is $T$ a proper subset of $S$? Explain.

(4) Suppose that every type of love can be expressed as the presence or absence of each of these 3 qualities: intimacy, passion, and commitment. How many different types of love are there. Describe each one.
(5) You might have to refer to your old calculus book to answer this question. In calculus, you learned about convergence and divergence of infinite series. You were given the definition of convergence and some theorems. One theorem was called the comparison test. Let

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

and

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$ 

Use the comparison test to prove that $\sum_{n=1}^{\infty} a_n$ converges by comparing it to $\sum_{n=1}^{\infty} b_n$. You may assume that $\sum_{n=1}^{\infty} b_n$ converges.

Proof. .

(6) Read an article from *Math Horizons*, published by the Mathematical Association of America, issues November 2004 through November 2006. On an attached sheet, give a summary of the article in 500 words or less. I have the issues of the journal from which you may copy an article. For more information, see http://www.maa.org/mathhorizons/.