10. Raul and Karli want to divide a chocolate-strawberry mousse cake. Raul values chocolate three times as much as he values strawberry. Karli values chocolate twice as much as she values strawberry.

Chocolate \[3 = \frac{3}{4}\]  
Raul’s eyes

Strawberry \[1 = \frac{1}{4}\]

Chocolate \[2 = \frac{2}{3}\]  
Karli’s eyes

Strawberry \[1 = \frac{1}{3}\]

a.) If Raul is the divider, which of the following cuts are consistent with Raul’s value system?

Cut 1  
Cut 2  
Cut 3  
60°
Cut 1: \( \frac{1}{4} (.5) + \frac{1}{4} (.5) = .5 \) Top
This would be a valid cut because each half is equal to 50% of the total cake in Raul’s eyes.

Cut 2: \( \frac{1}{4} (1) + \frac{1}{4} (0) = .75 \) Top
This is not a valid cut because the top is worth 75% in Raul’s eyes.

Cut 3: \( \frac{60}{180} = \frac{1}{3} \)
\( \frac{1}{4} (1) + \frac{3}{4} (1/3) = .5 \) Bottom
This is a valid cut because each half is equal to 50% of the total cake in Raul’s eyes.

Cut 4: \( \frac{60}{180} = \frac{1}{3} \)
\( \frac{72}{180} = \frac{2}{5} \)
\( \frac{1}{4} (2/5) + \frac{3}{4} (1/3) = .35 \)
This is not valid because the left cut is only worth 35% in Raul’s eyes.

Cut 5: \( \frac{72}{180} = \frac{2}{5} \)
\( \frac{96}{180} = \frac{8}{15} \)
\( \frac{1}{4} (2/5) + \frac{3}{4} (8/15) = .5 \) to the left
This is a valid cut because each half is equal to 50% of the total cake in Raul’s eyes.

b.) For each of the cuts consistent with Raul’s value system, indicate which of the pieces is Karli’s best choice.

Cut 1:
\( \frac{2}{3} (.5) + \frac{1}{3} (.5) = .5 \) top and bottom
Karli would pick either one because they are both equal in her eyes.

Cut 3:
\( \frac{2}{3} (1/3) + \frac{1}{3} (1) = .56 \) bottom
Karli would pick the bottom because it is worth the most, 56%, of the cake in her eyes.

Cut 5:
\( \frac{1}{3} (2/5) + \frac{2}{3} (8/15) = .49 \) to the left
Karli would pick the right side because it is worth the most, 51%, of the cake in her eyes.
Three heirs (Andre, Bea, and Chad) wish to divide up an estate consisting of a house, a small farm, and a painting, using the method of sealed bids. The heirs’ bids on each of the items are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>ANDRE</th>
<th>BEA</th>
<th>CHAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOUSE</td>
<td>$150,000</td>
<td>$146,000</td>
<td>$175,000</td>
</tr>
<tr>
<td>FARM</td>
<td>$430,000</td>
<td>425,000</td>
<td>428,000</td>
</tr>
<tr>
<td>PAINTING</td>
<td>50,000</td>
<td>59,000</td>
<td>57,000</td>
</tr>
</tbody>
</table>

Describe the final outcome of this fair-division problem.

Ok, let’s break this down into steps. **Step 1 (Bidding):** Each of the players have to make a bid for each of the items in the estate. (Note: it is very important that the players are honest with their bids, giving an assessment of the dollar value of each item.) (Note: it is also important to keep these bids private. The easiest way to do this is for each player to submit his or her bid in sealed envelopes.) The above table is a result of the completion of the bids. **Step 2 (Allocation):** Each item goes to the highest bidder for that item. In this case, the house would go to Chad, the farm would go to Andre, and the painting would go to Bea. **Step 3 (Payments):** Depending on what items (if any) a player gets in Step 2, he or she will owe money to or be owed money by the estate. To determine how much is owed, we first calculate how much each player believes his or her fair share is worth. This is done by adding the player’s bids and dividing by the number of players. If the total value of the items that the player gets in Step 2 is more than the value of that
player’s fair share, the player pays the estate the difference. If the total value of the items that the player gets is less than the value of the player’s fair share, the player collects the difference in cash.

<table>
<thead>
<tr>
<th></th>
<th>Andre</th>
<th>Bea</th>
<th>Chad</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOUSE</td>
<td>$150,000</td>
<td>$146,000</td>
<td>$175,000</td>
</tr>
<tr>
<td>FARM</td>
<td>$430,000</td>
<td>$425,000</td>
<td>$428,000</td>
</tr>
<tr>
<td>PAINTING</td>
<td>$50,000</td>
<td>$59,000</td>
<td>$57,000</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$630,000</td>
<td>$630,000</td>
<td>$660,000</td>
</tr>
<tr>
<td>FAIR SHARE</td>
<td>$210,000</td>
<td>$210,000</td>
<td>$220,000</td>
</tr>
</tbody>
</table>

Now we see who gets what. Chad gets the house because he was the highest bidder. Andre gets the farm because he was the highest bidder. The painting goes to Bea because she had the highest bid.

Now we figure out who owe’s the estate money, and who the estate owe’s money to. Chad got the house for $175,000, and his fair share is $220,000. This means that he didn’t receive his fair share from the deal yet, so the estate has to pay him the difference! $220,000-175,000=45,000. This means that Chad receives $45,000 from the estate! Andre got the farm for $430,000, but his fair share was only $210,000. This means that he owe’s the estate the difference. So Andre will pay the estate $210,000-$430,000= $-220,000. So Andre will pay the estate $220,000. Bea got the painting for $59,000, and her fair share is $210,000. This means that the estate owe’s her the difference. 210,000-59,000= 151,000. So the estate will give Bea $151,000.
Seems like we're all done right? Wrong. In this case there is a Step 4 (Dividing the Surplus): this is where we will divide the surplus evenly among the players. But first let's see how we even found out there was extra money. We see that the money that is being payed to the estate is $220,000. The money that the estate payed out is only $196,000. So you figure out the difference, $220,000 - $196,000 = $24,000. This means that there is $24,000 extra! This becomes a bonus, and will make each player even happier with the turnout. Since there is 3 players, we divide the $24,000 by 3, and we get $8,000. This means that each player will get an extra $8,000!

Let's take a look at the finished product!

<table>
<thead>
<tr>
<th>Chad</th>
<th>$175,000 House, $45,000, $8,000 Bonus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andre</td>
<td>$430,000 Farm, $-220,000, $8,000 Bonus</td>
</tr>
<tr>
<td>Bea</td>
<td>$59,000 Painting, $210,000, $8,000 Bonus</td>
</tr>
</tbody>
</table>

(Note: The sealed bids method only works if, #1, each player is honest, and has the money that he is bidding with, and #2, each player must accept money in substitute for any item (Items can not be priceless to any player).)