(1a) \[ \sum_{k=1}^{n} (2k+1) = n(n+2) \]

Proof: \[ \sum_{k=1}^{n} (2k+1) = \sum_{k=1}^{n} 2k + \sum_{k=1}^{n} 1 \]
\[ = 2 \sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 \]
\[ = 2 \left( \frac{n(n+1)}{2} \right) + n \]
\[ = n(n+1) + n \]
\[ = n(n+2) \]

(2) \[ \sum_{i=1}^{n} (-1)^i i^2 = (-1)^n \frac{n(n+1)}{2} \]

Proof: Base case: \[ \text{LHS} = \sum_{i=1}^{n} (-1)^i i^2 = (-1)^1 1^2 = -1 \]
\[ \text{RHS} = (-1)^1 \left( \frac{1(2)}{2} \right) = -1 \]

Inductive hypothesis: Let \( k \geq 1 \) and assume:
\[ \sum_{i=1}^{k} (-1)^i i^2 = (-1)^k \frac{k(k+1)}{2} \]

Next we show:
\[ \sum_{i=1}^{k+1} (-1)^i i^2 = (-1)^{k+1} \frac{(k+1)(k+2)}{2} \]
\[ \text{LHS} = \sum_{i=1}^{k+1} (-1)^i \cdot i^2 = \sum_{i=1}^{k} (-1)^i i^2 + (-1)^{k+1} (k+1)^2 \]

\[ = (-1)^k \left( \frac{k(k+1)}{2} \right) + (-1)^{k+1} (k+1)^2 \]

\[ = (-1)^k (k+1) \left( \frac{k}{2} + (-1)(k+1) \right) \]

\[ = (-1)^k (k+1) \left( \frac{k-2(k+1)}{2} \right) = (-1)^k (k+1) \left( \frac{-k-2}{2} \right) \]

\[ = \frac{(-1)^k (k+1)(-1)(k+2)}{2^2} = (-1)^k \frac{(k+1)(k+2)}{2} \]

\[ \sum \]

\[ 2^n \leq n! , \quad n \in \mathbb{N}, \quad n \geq 4. \]

**Pf:** Base Case, \( n = 4 \)

\[ 2^4 = 16 \]

\[ 4! = 24 \]

\[ 16 \leq 24 \], so the base case holds.

Assume \( k \geq 4 \) and \( 2^k \leq k! \) \( (1) \)

We know \( 2 \leq k+1 \)

Multiply \( (1) \) on left by 2 and on right by \( k+1 \)

We get

\[ 2 \cdot 2^k \leq (k+1) k! \]

\[ 2^{k+1} \leq (k+1)! \]
(9g) \[(A C) \implies B\] should be false.

pf: Let \(x = \frac{3}{2}\).

then \(A\) is true.
So \(A C\) is true.
But \(B\) does not hold.

Prop1: You supply reasons. Remember 1 axiom or reason per step.

(2) \(x \cdot 0 = 0\)

pf: \[0 + 0 = 0\]
\[x(0 + 0) = x \cdot 0\]
\[x \cdot 0 + x \cdot 0 = x \cdot 0\]
\[(x \cdot 0 + x \cdot 0) + -(x \cdot 0) = x \cdot 0 + -(x \cdot 0)\]
\[x \cdot 0 + (x \cdot 0 + -(x \cdot 0)) = x \cdot 0 + -(x \cdot 0)\]
\[x \cdot 0 + 0 = 0\]
\[x \cdot 0 = 0\]

(3) \((-x)y = -(xy)\).

pf: To show this, we will show \((-x)y\) is the additive inverse of \(xy\). Since inverses are unique, that implies that \((-x)y = -(xy)\).

We will show \(xy + (-x)y = 0\)

LHS = \(xy + (-x)y\)
\[= x(y + (-x)y)\]
\[= 0 \cdot y\]
\[= 0\]
(4) \(-x = (-1)x\)

pf: By (3) we have
\((-s)t = -(st)\) \(*

Letting \(s=1\) and \(t=x\)
we get
\((-1)x = -(1.x)\)
and since \(1.x = x\), we have
\((-1)x = -x\).

(5) \((-x)(-y) = xy\)

pf: First show \(z\) is the additive inverse of \(-z\), that is, \(z = -(-z)\).
\(-z + z = z + -z = 0\)

Using \(*\), let \(s = x\) and \(t = -y\).
we have:
\((-x)(-y) = -(x(-y))\) \(*\ *

Using Prop. 3 with roles of \(x\) and \(y\) reversed, we have,
\(x(-y) = -(xy)\)
so substitution in \(*\ *\) we have,
\((-x)(-y) = -(-(xy))\)
Now by \(!\)
\((-x)(-y) = xy\).
(4) \( q \in \mathbb{R}, q \neq 1, n \in \mathbb{N}, \)

\[
\sum_{i=0}^{n-1} q^i = \frac{q^n - 1}{q - 1}
\]

**Pf:**  
**Base case:** \( n = 1 \)

\[
\text{LHS} = \sum_{i=0}^{0} q^i = q^0 = 1
\]

\[
\text{RHS} = \frac{q^1 - 1}{q - 1} = 1
\]

\( \therefore \) Base case holds.

**Assume** \( k \geq 1 \) and \( \sum_{i=0}^{k-1} q^i = \frac{q^k - 1}{q - 1} \)

We will show: \( \sum_{i=0}^{k+1} q^i = \frac{q^{k+1} - 1}{q - 1} \)

\[
\text{LHS} = \sum_{i=0}^{k} q^i = \sum_{i=0}^{k-1} q^i + q^k
\]

\[
= \frac{q^{k-1}}{q - 1} + q^k = \frac{q^k - 1 + q^{k+1}}{q - 1}
\]

\[
= \frac{q^{k+1} - 1}{q - 1}
\]

\( \therefore \) Base case holds.