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MTH 307
2.2 #4
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Show that the sum of the cubes of any three consecutive natural numbers is a multiple of 9.

Let k and n be integers.

$$n^3 + (n+1)^3 + (n+2)^3 = 9k$$

Base Case;

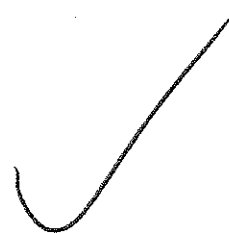
Let $n=1$

$$1^3 + 2^3 + 3^3 = 9j$$

$$36 = 9j$$

$$4 = j$$

4 is an integer, therefore S1 is true.



Now a k greater than one needs to be shown true so that the base case can show implication to any other case.

Assume that $n^3 + (n+1)^3 + (n+2)^3 = 9k$ is true for any value of k .

Show for an integer j

$$(n+1)^3 + (n+2)^3 + (n+3)^3 = 9j$$

Expand $(n+3)^3$ into $n^3 + 9n^2 + 27n + 27$

$$(n+1)^3 + (n+2)^3 + n^3 + 9n^2 + 27n + 27 = 9j$$

Notice that $(n+1)^3 + (n+2)^3 + n^3$ is already proven equal to $9k$.

$$9k + 9n^2 + 27n + 27 = 9j$$

Factor 9 out of both sides

$$9(k + n^2 + 3n + 3) = 9j$$

$$\text{Therefore } j = k + n^2 + 3n + 3$$

Since k and n are both integers, j is an integer so it is true in all cases by the induction principle.