+4

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2.2 #4
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Show that the sum of the cubes of any three consecutive natural numbers is a multiple of 9.

Let k and n be integers.

$n^3 + (n+1)^3 + (n+2)^3 = 9k$

Base Case;
Let n=1
$1^3 + 2^3 + 3^3 = 9j$
$36=9j$
$4=j$
4 is an integer, therefore S1 is true.

Now a k greater than one needs to be shown true so that the base case can show implication to any other case.

Assume that $n^3 + (n+1)^3 + (n+2)^3 = 9k$ is true for any value of k.

Show for an integer j

$(n+1)^3 + (n+2)^3 + (n+3)^3 = 9j$

Expand $(n+3)^3$ into $n^3 + 9n^2 + 27n + 27$

$(n+1)^3 + (n+2)^3 + n^3 + 9n^2 + 27n + 27= 9j$

Notice that $(n+1)^3 + (n+2)^3 + n^3$ is already proven equal to 9k.

$9k + 9n^2 + 27n + 27= 9j$

Factor 9 out of both sides

$9(k + n^2 + 3n + 3)= 9j$

Therefore $j = k + n^2 + 3n + 3$

Since k and n are both integers, j is an integer so it is true in all cases by the induction principle.