\[
\begin{array}{c|c|c|c|c}
\text{P} & \text{G} & \sim \text{G} & \text{P} \land \sim \text{G} & \text{P} \Rightarrow \text{G} \\
\hline
\text{T} & \text{T} & \text{F} & \text{F} & \text{T} \\
\text{T} & \text{T} & \text{F} & \text{T} & \text{F} \\
\text{F} & \text{T} & \text{F} & \text{F} & \text{T} \\
\text{F} & \text{T} & \text{T} & \text{F} & \text{F} \\
\text{F} & \text{T} & \text{T} & \text{F} & \text{F} \\
\end{array}
\]

Both cols have the same truth value.

(2) \[\neg P = \forall M \in \mathbb{R}, \exists n \in E, |n| > M.\]

P is false; the negation of P is true.

To prove the negation of P is true, we let \( M \in \mathbb{R} \) be arbitrary. Choose \( n \) to be an even number greater than \( M \).

Then \(|n| > M| \).
(3) \[
\begin{array}{c|c|c|c|c|c}
P & Q & \neg P & \neg Q & \neg Q \Rightarrow \neg P & P \Rightarrow Q \\
\hline
T & T & F & F & T & T \\
T & F & F & T & F & T \\
F & T & T & F & T & T \\
F & F & T & T & T & T \\
\end{array}
\]

These 2 cols. have the same sequence of truthvalues.

(4) (a) Not a partition
Since not mutually disjoint.
For instance 2 \in N_1 \cap N_2.

(b) Yes, \( \mathcal{Q} \) is a partition

(c) Yes, \( \mathcal{D} \) is a partition

(d) \( \mathcal{D} \) is coarser than \( \mathcal{Q} \)
\( N_0 \subseteq D_0, N_1 \subseteq D_1, N_2 \subseteq D_2, N_3 \subseteq D_0, N_4 \subseteq D_1, N_5 \subseteq D_2 \)

(5) (a) \{0, 1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\} \cup \{1, 2, 3, 4, 5\}

(b) \{0\} \cup \{1, 2\} \cup \{2, 3\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\} \cup \{1, 2, 3, 4, 5\}

(c) \( \mathbb{Z} \)
(6) Let \((x, y) \in (A \cap B) \times C\)

then \(x \in A \cap B\) and \(y \in C\), by def of cross product.

So \(x \in A\) and \(x \in B\), by def of \(\cap\).

Thus, \((x, y) \in A \times C\), by def of cross product

and \((x, y) \in B \times C\), by def of cross product.

Thus, \((x, y) \in (A \times C) \cap (B \times C)\), by def of \(\cap\).

(7)

\[3 \times 2 \times 3 = 18\]

(8) Let \(x \in A \cup (B \cap C)\)

then \(x \in A\) or \(x \in B \cap C\).

Case 1: \(x \in A\)

In this case, \(x \in A \cup B\) and \(x \in A \cup C\) by def of \(\cup\)

so \(x \in (A \cup B) \cap (A \cup C)\) by def of \(\cap\).

Case 2: \(x \notin A\)

So \(x \in B \cap C\). Then \(x \in A \cup B\) and \(x \in A \cup C\) by def of \(\cup\)

and \(x \in (A \cup B) \cap (A \cup C)\) by def of \(\cap\).
\( \begin{array}{c|cccc|c}
P & Q & \neg P & \neg Q & \neg P \lor \neg Q & P \Rightarrow Q \\
--- & --- & --- & --- & --- & --- \\
T & T & F & F & F & T \\
T & F & F & T & F & T \\
F & T & T & F & T & T \\
F & F & T & T & T & T \\
\end{array} \)

These cells have the same truth values.

\((10)\) \(\forall m \in \mathbb{Z}, \exists k \text{ such that } M = 2k - 1\)

We use \(2n+1\) \(n \in \mathbb{Z}\) to be the defn of odd.

This statement is true.

Proof: Let \(m \in \mathbb{Z}\).

Then for some \(n \in \mathbb{Z}\), \(m = 2n+1\)

Let \(k = n+1\), so \(n = k - 1\)

Then \(M = 2(k-1)+1 = 2k-2+1 = 2k-1\)

Hence \(m \in 2k-1\) for some \(k \in \mathbb{Z}\).

\((11)\) Negation: \(\exists a, b \in \mathbb{R}, a \neq b \text{ and } a^2 - 7 = b^2 - 7\).

The negation is true.

For instance, let \(a = 2\) and \(b = -2\)

then \(a \neq b \text{ and } a^2 - 7 = b^2 - 7\).
(12) Negation: \( \exists x \in \mathbb{R}, \forall n \in \mathbb{N}, x > n \).

This is False.
For every real \( x \), there is a natural number at least as large as \( x \).

(13) Negation: \( \forall n \in \mathbb{N}, \exists x \in \mathbb{R}, x > n \).

This is true.
Let \( n \in \mathbb{N} \), choose \( x = n + 1 \).

(14) (a) \( \forall x \in \mathbb{R}, \text{ if } x \neq 0 \text{ then } ax \neq 0 \)
(b) \( \forall x \in \mathbb{R}, \text{ if } x = 0 \text{ then } ax = 0 \)
(c) \( \exists x \in \mathbb{R}, \text{ such that } ax = 0 \text{ and } x \neq 0 \).
(d) False. If \( a = 0 \), we see that \( x \) could equal \( 1 \) and we would have \( x \neq 0 \) but \( ax = 0 \).
(e) True. If \( x = 0 \), then \( ax = 0 \) for all \( a \).

(15) Negation: \( \forall x \in \mathbb{R}, x < 100 \)

False

(16) \( \forall n \in \mathbb{E}, \exists a \text{ prime number } p \exists b \text{ an even number such that } n = p - b \).

False. Let \( n = 2 \).
(No number)
(a) \( D_1 = \{1, 3\} \), \( D_2 = \{1, 2, 3\} \), \( D_6 = \{1, 2, 3, 6\} \)
(b) \( |D_1| = 1 \), \( |D_2| = 2 \), \( |D_6| = 4 \)
(c) \( \mathcal{P}(D_{10}) = \{\emptyset, \{1, 3\}, \{3, 5, 6\}, \{1, 5, 6\}, \{3, 10, 13\}, \{1, 2, 3, 5, 13\}, \{1, 3, 10, 13\}, \{3, 5, 13\}, \{2, 5, 13\}, \{2, 10, 13\}, \{5, 13\}, \{1, 2, 5, 10, 13\} \)

(17)
(a) \( 0, 6, 12 \)
(b) \( (0, 3), (0, 6), (0, 12) \)

(18) Let \( (n, \infty) \) and \( (m, \infty) \) be two arbitrary elements of \( \mathcal{C} \), where \( n \neq m \).

Then \( n, m \in \mathbb{Z} \). Assume without loss of generality that \( n < m \).

Then \((m, \infty) \subseteq (n, \infty)\).

Therefore, \( \mathcal{C} \) is nested.