MTH 141 Notes 3.1

Theorem 3.1 If f is differentiable and c is a constant, then $\frac{d}{dx}\left[c \cdot f(x)\right] = c \cdot f'(x)$.

Proof:

Theorem 3.2 If f and g are differentiable, then $\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$.

Proof:

The Power Rule: For any constant real number n, $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$.

Proof:

Example 1: Use the power rule to differentiate **(a)** $\frac{1}{x^3}$, **(b)** $\sqrt{x}$, **(c)** $\frac{1}{\sqrt[3]{x}}$.

Example 2: Use the definition of the derivative to justify the power rule for $n = -2$.

Example 3: Find the derivatives of **(a)** $6x^2 - 3x + 4$ and **(b)** $\sqrt{3}x^7 - \frac{x^5}{5} + \pi$.

Example 4: Differentiate **(a)** $5\sqrt{x} - \frac{10}{x^2} + \frac{1}{2\sqrt{x}}$ and **(b)** $0.1x^3 + 2x^{\sqrt{2}}$.

Example 5: Find the second derivative and interpret the sign for:
(a) $f(x) = x^2$ **(b)** $g(x) = x^3$ **(c)** $k(x) = \sqrt{x}$

Example 6: If the position of an object, in meters, is given as a function of time *t*, in seconds, by $s(t) = -4.9t^2 + 5t + 6$, find the velocity and acceleration of the object at time *t*.

Example 7: *f* is a cubic polynomial. Both *f* and f' are graphed below. Both graphically and algebraically, describe the behavior of the derivative of this cubic.