MTH 132

Quiz IV

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Show all your work.!

(1) Toxins in pesticides can get into the food chain and accumulate in the body. A person consumes 10 micrograms a day of a toxin, ingested throughout the day. The toxin leaves the body at a continuous rate of 3% everyday. Write a differential equation for the amount of toxin, A, in micrograms, in the person's body as a function of the number of days, t.

$$\frac{dA}{dt} = 10 - 0.03A$$

(2) The volume and outflow of Lake Michigan and Lake Ontario are respectively

Michigan	4900 km^3	158 (km^3/year)
Ontario	1600 km^3	209 (km^3/year)

Find how long it would take for 75% of the pollution to be removed from Lake Michigan and from Lake Ontario, assuming no new pollutants are added and the rate of outflow is the same as the rate at which new water flows into the lakes.

Pollution outflow = $\left(\frac{Q}{V}\right) * (-R)$

$$\frac{dQ}{dt} = \frac{-r}{V} Q$$

$$\int \frac{dQ}{Q} = \int \frac{-r}{V} dt$$

$$\ln Q = \frac{-r}{V} t + c$$

$$Q = c e^{(-r/V)t}$$

Lake Michigan

$$0.25 Q_0 = Q_0 e^{\left(\frac{-158}{4900}\right)t}$$

$$0.25 = e^{-.032245 t}$$

$$\ln(0.25) = -.032245 t$$

$$\frac{\ln(0.25)}{-0.032245} = t = 42.99 \text{ years } (\approx 43 \text{ years})$$

Lake Ontario

$$0.25 Q_0 = Q_0 e^{(-209/1600)t}$$

$$\frac{\ln 0.25}{-209/1600} \quad t = 10.61 \text{ years}$$

(3) Hydrocodone bitartrate is used as a cough suppressant. After the drug is fully absorbed, the quantity of drug in the body decreases at a rate proportional to the amount left in the body. The half-life of hydrocodone bitartrate in the body is 3.8 hours and the dose is 10 mg.

(a) Write an ODE for the quantity, Q, of hydrocodone bitartrate in the body at time t, in hours since the drug was fully absorbed.
(b) Solve the ODE found in the item above.
(c) Use the half-life to find the constant of proportionality, k
(d) How much of the 10 mg. dose is still in the body after 12 hours?

$\boxed{A.}$ $\frac{dQ}{dt} = -KQ$

$\boxed{B.}$ $\int \frac{dQ}{Q} = \int -K\,dt$

$\ln Q = -Kt + C$

$e^{\ln Q} = e^{-Kt+C}$

$Q = ce^{-Kt}$

$\boxed{C.}$ $0.5\cancel{C}e^{-K(0)} = \cancel{C}e^{-K(3.8)}$

$0.5 = e^{-3.8K}$

$\ln(0.5) = -3.8K$

$K = \frac{\ln(0.5)}{-3.8} = 0.1824 = K$

$\boxed{D.}$

$10 = ce^{-K(0)}$

$10 = C$

$Q = 10e^{(-.1824)(t)}$

$Q = 10e^{(-.1824)(12)}$

$Q = 1.12$ mg

(4) A bank account earns 5% annual interest compounded continuously. Continuous payments are made out of the account at a rate of \$12,000 per year for 20 years.

(a) Write an ODE describing the balance $B = f(t)$, where t is in years.

(b) Solve the ODE given an initial balance of B_0

(c) What should the initial balance be such that the account has balance zero after precisely 20 years?

$\boxed{A}$ $\frac{dB}{dt} = 0.05B - 12{,}000$

$\boxed{B}$ $\int \frac{dB}{0.05B - 12{,}000} = \int dt$

$u = 0.05B - 12{,}000$

$du = 0.05\,dB$

$20\,du = dB$

$20 \int \frac{du}{u} = \int dt$

$20 \ln|u| = t + c$

$20 \ln|0.05B - 12{,}000| = t + c$

$\ln|0.05B - 12{,}000| = \frac{t + c}{20}$

$0.05B - 12{,}000 = e^{\frac{t+c}{20}}$

$0.05B - 12{,}000 = C_1 e^{t/20}$

$0.05B = C_1 e^{t/20} + 12{,}000$

$B = \frac{C_1 e^{t/20}}{0.05} + \frac{12{,}000}{0.05}$

$B = C_2 e^{0.05t} + 240{,}000$

$B_0 = C_2 e^{0.05(0)} + 240{,}000$

$B_0 - 240{,}000 = C_2$

$B = (B_0 - 240{,}000)e^{0.05t} + 240{,}000$

Check:

$\frac{dB}{dt} = 0.05B - 12{,}000$

$\frac{dB}{dt} = 0.05(B - 240{,}000)$

$B = Ce^{0.05t} + 240{,}000$

$\boxed{C}$

$0 = (B_0 - 240{,}000)e^{(0.05)(20)} + 240{,}000$

$-240{,}000 + 240{,}000e = B_0 e$

$\frac{-240{,}000}{e} + 240{,}000 = B_0 = \$151{,}708.93 \to \approx \$151{,}709$