Show all your work.!

(1) Determine whether \( f(z) \) is analytic or not. If \( f \) is analytic, find the domain of analyticity of \( f(z) \) and compute \( f'(z) \).

(a) \( f(z) = e^z = \exp(z) \)
(b) \( f(z) = 1/z \)
(c) \( f(z) = x^2 + iy^2 \)
(d) \( f(z) = \begin{cases} \frac{\pi}{z} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases} \)
(2) If

\[
\frac{\partial f}{\partial z} = \frac{1}{2} \left( \frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y} \right)
\]

\[
\frac{\partial f}{\partial \bar{z}} = \frac{1}{2} \left( \frac{\partial f}{\partial x} + i \frac{\partial f}{\partial y} \right)
\]

Show that a harmonic function \( u \), satisfies the formal differential equation

\[
\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0
\]
(3) Find an analytic function $f$, whose real part is given by

$$u(x, y) = (e^x + e^{-x}) \cos y$$
(4) Find all the roots of the equation
\[
\sin z = 2
\]
(5) Show that
(a) \((-1 + \sqrt{3}i)^{3/2} = \pm 2\sqrt{2}\)
(b) \((1 + i)^i = \exp(-\frac{\pi}{4} + 2n\pi) \exp(\frac{i}{2 \ln 2}), \quad n \in \mathbb{Z} \).

Note \(e^x = \exp(x)\).
(6) Show that

(a) the set of values of $\log(i^{1/2})$ is $(n + \frac{1}{4})\pi i$ ($n = 0, \pm 1, \pm 2, \ldots$) and that the same is true of $(1/2)\log i$;

(b) the set of values of $\log(i^2)$ is not the same as the set of values of $2\log i$. 
Extra Credit for Exam.
(7) Show that
(a) the function \(\text{Log}(z - i)\) is analytic everywhere except on the half line \(y = 1, \quad (x \leq 0)\);
(b) the function
\[
\frac{\text{Log}(z + 4)}{z^2 + i}
\]
is analytic everywhere except at the points \(\pm(1 - i)/\sqrt{2}\) and on the portion \(x \leq -4\) of the real axis.

(Note. Here \(\text{Log}(z)\) denotes the principal value of \(\log z\).)