MTH 215

Quiz IV

Name:

Show all your work.!

(1) Compute the determinant by cofactor expansions. At each step choose the row or column that involves the least amount of computation.

$$\begin{vmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

(2) Let $\mathbf{u} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$. Compute the area of the parallelogram determined by $\mathbf{u}$, $\mathbf{v}$, $\mathbf{u}+\mathbf{v}$ and $\mathbf{0}$.

(3) Find the determinants, where

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7.$$

(a) $\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix}$

(b) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix}$

(c) $\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix}$

(4) Use **determinants** to decide if the set of vectors is linearly independent

$$\begin{pmatrix} 3 \\ 5 \\ -6 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ -6 \\ 0 \\ 7 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ -1 \\ 3 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3 \end{pmatrix}$$

(5) Let H be the set of points inside and on the unit circle in the xy-plane. That is

$$H = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x^2 + y^2 \leq 1 \right\}.$$

Show that H is **not** a subspace of $\mathbb{R}^2$.

What are the properties that a subset H of a vector space V should satisfy to be a subspace?

(6) Find a basis for Nul(A) and Col(A) by listing vectors that span the null space of A and the column space of A respectively. where

$$A = \begin{pmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Find

(a) s such that Nul(A) is a subspace of $\mathbb{R}^s$

(b) t such that Col(A) is a subspace of $\mathbb{R}^t$