MTH 215

EXAM II

November 12, 2004

DUE NOVEMBER 15, 2004 at 3:00 PM

Name:

Show all your work

(1) Find the standard matrix representation A for the reflection of the plane in the line $5x - 2y = 0$.

(2) (a) Let R be the parallelogram determined by the vectors $\vec{b}_1 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\vec{b}_2 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$, and let $A = \begin{pmatrix} 6 & -2 \\ -3 & 2 \end{pmatrix}$. Compute the area of the image of R under the mapping $\vec{x} \mapsto A\vec{x}$.
(In other words compute the area of the parallelogram $A(R)$).

(b) Find the volume of the parallelepiped (box) with one vertex at the origin and adjacent vertices at $(1, 0, -2)$, $(1, 2, 4)$, $(7, 1, 0)$.

(3) (a) Compute $\det B^5$, where $B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$

(b) Suppose A is a square matrix such that $\det A^4 = 0$. Explain why A can not be invertible.

(c) Compute the determinant of the following matrix. **Make sure that you do it in as FEW steps as possible!**

$$\begin{pmatrix} 1 & -3 & 1 & -2 \\ 2 & -5 & -1 & -2 \\ 0 & -4 & 5 & 1 \\ -3 & 10 & -6 & 8 \end{pmatrix}$$

(4) Find the adjoint of matrix A and use it to find A^{-1}.

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 4 & -2 \end{pmatrix}$$

(5) Given the matrix

$$A = \begin{pmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{pmatrix}$$

(a) Find the characteristic polynomial of A.

(b) Find **all** the eigenvalues of A.

(c) Find a basis for the eigenspace corresponding to each eigenvalue in (b).

(6) Use Cramer's rule to compute the solution of the system

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 4 \\ -x_1 + \phantom{x_2 +{}} 2x_3 &= 2 \\ 3x_1 + x_2 + 3x_3 &= -2 \end{aligned}$$

(7) Assume that T is a linear transformation.

(a) Find the standard matrix representation of T when $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a "horizontal shear " transformation that maps $\hat{e_2}$ into $\hat{e_2} - 3\hat{e_1}$ but leaves the vector $\hat{e_1}$ unchanged.

(b) Find the standard matrix representation of T when $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points clockwise about the origin through an angle of $\frac{3}{4}\pi$ radians.

(8) (a) Find the values of λ for which the given matrix is singular

$$\begin{pmatrix} 2-\lambda & 0 & 0 \\ 0 & 1-\lambda & 4 \\ 0 & 1 & 1-\lambda \end{pmatrix}$$

(b) Let

$$A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$$

Find the **all** eigenvalues of A. One of the eigenvalues of A is zero. Based on this fact, what can you conclude about A. **Justify your answer**.