Name: ______________________________

MTH 243 *Quiz 4* *April 3, 2014*

Explain your reasoning clearly. Show all steps.

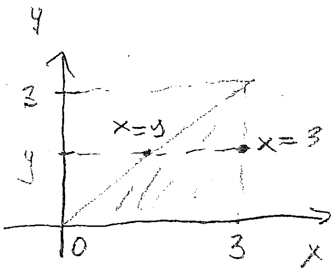
1. For the integral

$$\int_0^3 \int_y^3 \sin(x^2)\,dx\,dy$$

(a) Sketch the region of integration.

(b) Reverse the order of integration and evaluate the integral.

(a)



$$\int_0^3 \int_y^3 \sin(x^2)\,dx\,dy = \int_0^3 \int_0^x \sin(x^2)\,dy\,dx =$$

Antiderivative cannot be found

$$= \int_0^3 x \sin x^2\,dx = -\tfrac{1}{2}\cos(x^2)\Big|_0^3 = -\tfrac{1}{2}\cos(9) + \tfrac{1}{2} =$$

$$\cong 0.96$$

Mode radians!

2. Let $f(x, y) = x^3 + y^3 - 6y^2 - 3x + 9$.

(a) Find all critical points of $f(x, y)$.

(b) Classify each critical point as a local minimum, a local maximum, or a saddle point.

$f_x = 3x^2 - 3$, $f_y = 3y^2 - 12y$ Cr. pts: $\begin{cases} 3x^2 - 3 = 0 & \to x = \pm 1 \\ 3y^2 - 12y = 0 & \to y = 0, 4 \end{cases}$

Cr pts: $(1,0)$ $(1,4)$, $(-1,0)$, $(-1,4)$.

$f_{xx} = 6x$, $f_{yy} = 6y - 12$, $f_{xy} = 0$. $D(x,y) = 6x(6y - 12)$

$D(1,0) = 6(-12) < 0$ – saddle, $D(-1,4) = -6(6\cdot 4 - 12) < 0$ saddle

$D(-1,0) = -6(-12) > 0$ $f_{xx}(-1,0) = -6 < 0$ loc. max

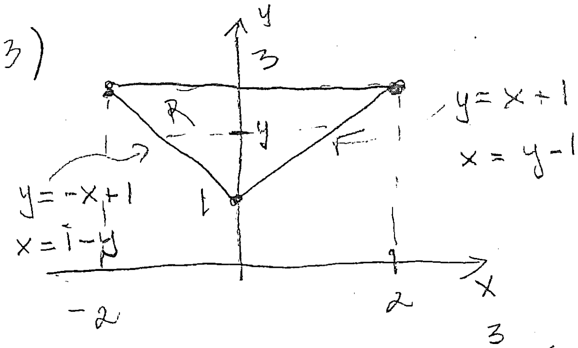
$D(1,4) = 6(24-12) > 0$ $f_{xx}(1,4) = 6 > 0$ loc. min.

3. Compute the double integral:

$$\int_R (2x^2 + y)\,dA$$

where R is the triangle with vertices $(0, 1), (-2, 3), (2, 3)$. Be sure to sketch the region of integration and carefully set up an iterated integral.
(Use the other side of this sheet for your solution.)

3)



$$\iint_R (2x^2+y)\,dA = \int_1^3 \int_{1-y}^{y-1} (2x^2+y)\,dx\,dy =$$

$$= \int_1^3 \left(\left(\frac{2}{3}x^3 + yx\right)\Big|_{x=1-y}^{x=y-1} \right) dy =$$

$$= \int_1^3 \left(\frac{2}{3}(y-1)^3 + (y-1)y - \frac{2}{3}(1-y)^3 - (1-y)y \right) dy =$$

$$= \int_1^3 \left(\frac{4}{3}(y-1)^3 + 2y(y-1) \right) dy = 14.6\overline{6}$$

Calculator

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$, $\frac{4}{3}(y^3 - 3y^2 + 3y - 1)$

$$\int_1^3 \left(\frac{4}{3}(y-1)^3 + 2y(y-1)\right) dy = \int_1^3 \left(\frac{4}{3}y^3 - 4y^2 + 4y - \frac{4}{3} + 2y^2 - 2y\right) dy =$$

$$= \int_1^3 \left(\frac{4}{3}y^3 - 2y^2 + 2y - \frac{4}{3}\right) dy =$$

$$= \left(\frac{1}{3}y^4 - \frac{2}{3}y^3 + y^2 - \frac{4}{3}y\right)\Big|_1^3 = (27 - 18 + 9 - 4) - \left(\frac{1}{3} - \frac{2}{3} + 1 - \frac{4}{3}\right) =$$

$$= 14 + \frac{2}{3} = 14.6\overline{6}$$