MTH 536 Final Exam -- Study Guide

Our Final Exam is scheduled for Thursday, May 10, 7-10 pm in Tyler 106. The exam is comprehensive. It covers classes 27-51 and homework assignments 1-7.

As always, you should know statements of all theorems, definitions, propositions given in class as well as most of their proofs. You should be familiar with all important counterexamples done in class. The following list contains problems that are good candidates for exam problems. As always, problems on the list may or may not appear on the exam, and problems that are not on the list may be on the exam. It is only a guide.

For the first part, Study Guide for Exam 1 applies. Here is a guide for the second part. (The material is listed backwards.)

(a) State the definition of an absolutely continuous function in [a,b].
(b) State basic properties of absolutely continuous functions (Th. 51.1, Th.51.2).
(c) Prove Prop.51.1.

Assume Lemma 49.1 and Lemma 49.2 as given. Prove Lemma 49.3. Prove Prop 50.1. Show how Prop 50.1 proves the Fubini Theorem for f(x,y) which is the characteristic function of a measurable subset E of $X \times Y$.

Let $(X, \mathcal{M}, \mu)$, $(Y, \mathcal{N}, \nu)$ be two complete measure spaces, $(X \times Y, \mathcal{S}, \mu \times \nu)$ the product space.
(a) State the Fubini Theorem.
(b) Give an example of a product space and a measurable subset E of $X \times Y$ such that not every cross section E_x is measurable in Y.

(a) For a subset E of $X \times Y$ and x in X, define the cross section E_x .

(b) Prove Lemma 48.1.

Let $(X,\mathcal{M}, \mu)$, $(Y,\mathcal{N}, \nu)$ be two complete measure spaces. Describe the construction of the product space $(X\times Y, \mathcal{S}, \mu\times\nu)$. (Without proofs.)

(a) State the definition of a semialgebra of subsets of a set X.
(b) Let $(X, \mathcal{M}, \mu)$, $(Y, \mathcal{N}, \nu)$ be two complete measure spaces. Define the collection of measurable rectangles, $\mathcal{R}$.
(c) Prove that $\mathcal{R}$ is a semialgebra. (That is, prove Prop. 46.1.)

Let X be a given nonempty set, $\mathcal{A}$ an algebra of subsets of X.
(a) State the definition of a measure, μ, on the algebra.
(b) State the definition of the induced outer measure μ^*.
(c) Prove Prop 44.1.

Let X be a given nonempty set, μ^* an outer measure on X.
(a) State the definition of regularity of the outer measure.
(b) Prove that the Lebesgue outer measure m^* in R is regular.

(a) State the definition of the Lebesgue outer measure m_k^* in R^k.
(b) Prove that m_k^* is regular. (You can assume known that Borel sets in R^k are measurable.)

Particularly good homework problems for H5-H7. (Some are already included in the list above):

H5: All;

H6: All;

H7: #4.