MTH 536 Exam 1 -- Study Guide

Exam is scheduled for Friday, March 30, 3-5 pm in Tyler 106. The exam covers classes 27-39 (including 39) and homework assignments 1-4. Problem 5 from Homework 5 (to be given out on Monday) may appear as it is a simple application of the MTC theorem and Fatou's Lemma.

As always, you should know statements of all theorems, definitions, propositions given in class as well as most of their proofs. You should be familiar with all important counterexamples done in class. The following list contains problems that are good candidates for exam problems. As always, problems on the list may or may not appear on the exam, and problems that are not on the list may be on the exam. It is only a guide. (The material is listed backwards.)

Prove Prop. 39.1;

Prove Prop. 39.2;

Prove Prop. 39.3;

(a) State the LDCT
(b) Prove the LDCT
(c) Prove something using the LDCT like H9 # 6 last semester or something that may look new.

(a) State the theorem about approximating a nonnegative measurable function by simple functions (Th. 36.2).
(b) Prove something simple using the theorem like Prop 37. 1 (a).

(a) State Fatou's Lemma.
(b) State the MCT.
(c) Prove the MCT
(b) Prove something like H5 #2 using the MCT.

(a) State the definition of completeness of a measure space.
(b) Prove Prop. 35.1
(c) Give an example that the proposition may not hold in a noncomplex measure space.

(a) State Egoroff's Theorem.
(b) Prove Th 35.2.

(a) State the definition of a measurable function.
(b) Prove something along the lines H3 #4 or H4 # 1 (a) or # 2.
(c) What functions are measurable in, say, $(X,\{X,0\})$, $(X,P(X))$.

(a) State the definition of a measure space.
(b) Prove Prop.32.4.

(a) State the definition of the totally σ–finite space.
(b) Give an example of a space which is not totally σ–finite.

Prove Prop. 32.1.

State and prove Prop. 32.3.

(a) Characterize convergence in $L^\infty([a,b])$. Prove your claim. (That is, prove Th.31.1.)

State and prove Hölder's inequality for p=1, q=∞. (That is, Prop 31.2.)

Define the essential supremum and prove Prop 30.2.

(a) State Hölder's inequality.
(b) Prove H1 # 4.

(a) State the definition of a Hilbert space.
(b) Explain why $L^2([a,b])$ is a Hilbert space.

(a) Show that the mapping $\| \circ \|_p$ is positively homogeneous.
(b) State the important theorem that gives additivity of the norm. (Minkowski's thing.)

(a) Define the Banach space C([a,b]).
(b) Show that its norm induces uniform convergence.

Particularly good homework problems:

H4: 1(a), 2, 3;

H3: 1, 2, 4;

H2: Definitely all;

H1: Definitely all.