MTH 435 Fall 2002 Homework 5, due: Oct 28

(Homework 1-5 are going to be included in Exam I).

1) Use the Triangle Inequality to show that

$$|x| - |y| \le |x - y|$$

for all $x, y \in \mathbb{R}$.

2) Use the Triangle Inequality and propositions proved in class to show the following:

Let $L, \varepsilon \in \mathbb{R}$, $\varepsilon > 0$. Let $a, b \in (L-\varepsilon, L+\varepsilon)$. Then

$$|a - b| < 2\varepsilon.$$

3) Show that the sequence $\{(-1)^n\}_{n=1}^{\infty}$ is divergent.

4) Is the sequence $\left\{\frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$ divergent or convergent? Prove your claim.

5) Show that

$$\lim_{n \to +\infty} \left(1 - \frac{1}{n^2}\right) = 1.$$

6) Suppose that the sequences $\{x_n\}_{n=1}^{\infty}$, $\{y_n\}_{n=1}^{\infty}$ are bounded. Show that the sequences

$$\{x_n + y_n\}_{n=1}^{\infty}, \quad \{x_n - y_n\}_{n=1}^{\infty}, \quad \{x_n y_n\}_{n=1}^{\infty}$$

are bounded.

7) Let $\{a_1, a_2, \ldots, a_k\}$ be a <u>finite</u> collection of real numbers, $a_i \in \mathbb{R}$ for $i = 1, \ldots, k$. Show that the collection has the largest element; that is, prove that there exists $a_{i_0} \in \{a_1, a_2, \ldots, a_k\}$ such that

$$a_i \le a_{i_0} \quad \text{for all } i = 1, 2, \ldots, k.$$

Hint: Use induction with respect to k.