MTH 435 **Final Exam - Tips** **Fall 2014**

Our Final Exam is scheduled for Friday, December 12, 3 pm - 6 pm, Lippitt 204. Final Exam is comprehensive and covers classes 1-37 and homework assignments 1-9. (Although, from Homework 9 only two problems are included. See below.)

For the material corresponding to Exam 1 see **Exam 1 Study Tips** linked on the course's page.

For the material corresponding to Exam 2 see **Exam 2 Study Tips** linked on the course's page.

Tips for Classes 30-37

You are expected to know and be able to state all definitions, theorems and propositions given in class. You should be familiar with all examples presented in class. Below are a few questions you should expect. They may appear in combination with statements of theorems or definitions. Below (X, d) is an arbitrary metric space.

- Prove that an open ball in X is open. (Th 31.1).

- Prove that the union of any family of open sets is open. (Th 31.3).

- Prove that the intersection of any finite family of open sets is open. (Th 31.4). Give an example that the intersection of an infinite family of open sets may not be open.

- Prove that a subset Y of X is closed if and only if its complement $X - Y$ is open. (Th 32.2)

- Show that any singleton set $\{x_0\}$, $x_0 \in X$ is closed.

- Give an example of a subset of $(\mathbf{R}, \mathrm{d})$ which is neither closed nor open. Prove your claims.

- What is the closure of the set of rationals $\overline{\mathbf{Q}}$ in $(\mathbf{R}, \mathrm{d})$? Prove your claim.

- Prove that the limit of a sequence $\{x_n\}_{n=1}^{+\infty}$ in X is unique. (Th 32.4)

- Prove that any convergent sequence $\{x_n\}_{n=1}^{+\infty}$ in X is Cauchy. (Th 32.4)

- Is the subspace $(\mathbf{Q}, \mathrm{d})$ of $(\mathbf{R}, \mathrm{d})$ complete? Prove your claim. (Th 33.2)

- Let $Y \subseteq X$. Prove that $x \in X$ is adherent to Y if and only if there is a sequence $\{y_n\}_{n=1}^{+\infty}$ such that $y_n \in Y$ and $\lim_{n\to+\infty} y_n = x$. (Th 1.11 page 7 in the Gamelin, Green book.)

- Prove that the set of rationals $\mathbf{Q}$ is countable. (Th 36.4)

- State the Nested Intervals Theorem (Th.36.6). Prove that $\mathbf{R}$ is not countable. (Th.36.5)

For Homework 1-8 see study guides for Exam 1 and 2. From Homework 9, go over problems #2 page 7, and #14 page 8. Also, read the proof of Th. 1.11 page 7.

Happy studying! I hope you all get As!