**MTH 435** **Exam 1 - Tips** **Fall 2014**

Exam 1 is scheduled for Friday Oct 17, 3-5 pm, Lippitt 204. The exam covers classes 1-16 and homework assignments 1-4.

You are expected to know and be able to state all definitions, theorems and propositions given in class. You are expected to know all proofs done in class. You are supposed to know solutions to homework problems. On the exam you will have to state some definitions, theorems and propositions given in class as well as prove things. Some proofs will come from those given in class. Some exam problems will be similar to homework problems. In addition, at least one problem not done in class and not similar to homework problems will appear. Below are a few sample questions of the kind you should expect.

- State the definition of the supremum and the infimum of a nonempty subset of $\mathbf{R}$.

- Prove that the supremum (or infimum) is unique.

- Find the supremum and the infimum of $A = \mathbf{Q} \cap (2,3)$. Prove your claims. Does $A$ have the largest element? The smallest element? Be sure to justify your claims.

- Define what it means that a set $S$, $S \subseteq \mathbf{R}$, is dense in $\mathbf{R}$.

- State and prove the theorem called Density of Rationals.

- Is the set of all integers $\mathbf{Z}$ dense in $\mathbf{R}$? Prove your claim.

- State the definition of a convergent sequence and its limit.

- Prove that the limit of a convergent sequence is unique.

- Show that the sequence $\{(-1)^n\}_{n=1}^{+\infty}$ diverges.

- Show from the definition of the limit that the sequence $\{\frac{2}{\sqrt{n}}\}_{n=1}^{+\infty}$ converges.

- Prove that if a sequence $\{a_n\}_{n=1}^{+\infty}$ is convergent, then it is bounded. Is the converse true? Justify your claim.

- Let

$$\lim_{n\to+\infty} a_n = L, \lim_{n\to+\infty} b_n = M,$$

where $M, L \in \mathbf{R}$. Prove that

$$\lim_{n\to+\infty} (a_n + b_n) = L + M$$

Or, for example, prove:

$$\lim_{n\to+\infty} (a_n \cdot b_n) = L \cdot M$$

- State and prove the Squeeze Theorem.

- State the definition of a Cauchy sequence.

- Prove that every convergent sequence is Cauchy.

- Show that the sequence $\{(-1)^n\}_{n=1}^{+\infty}$ is not Cauchy.

- State the Bolzano-Weierstrass Theorem. Find a convergent subsequence of the sequence $\{(-1)^n\}_{n=1}^{+\infty}$.

Homework problems that strike me as particularly good candidates to possibly include in the exam are: Homework 1: number 3; Homework 2: numbers 2(b), 5; Homework 3: numbers 2, 3, 4, 5; Homework 4: numbers 1,2,4 and 6.

Happy studying! I hope you all get As!