MTH 435 Homework 1 - Answers - Solutions - F14

1) (a) Equivalent (b) Equivalent (c) Not equivalent

2) (a) A tautology (b) Not a tautology (c) Not a tautology
(d) A tautology (e) A tautology

3) (a) False. If $x<0$, $x\cdot y \geq 0$ requires $y \leq 0$. But then $x+y<0$.
Negation: $\exists_{x\in\mathbb{R}} \forall_{y\in\mathbb{R}} (x\cdot y<0 \text{ or } x+y\leq 0)$.

(b) True. For a given x take $y=x$. Then $xyz^2 = x^2z^2 \geq 0$.
Negation: $\exists_{x\in\mathbb{R}} \forall_{y\in\mathbb{R}} \exists_{z\in\mathbb{R}} \; xyz^2<0$.

(c) True. Take $y=0$.
Negation: $\forall_{y\in\mathbb{R}} \exists_{x\in\mathbb{R}} \; x\cdot y \neq 0$

(d) True. Take $x=1$.
Negation: $\forall_{x\in\mathbb{R}} \exists_{y\in\mathbb{R}} (y>0 \text{ and } xy\leq 0)$

4) Denote open statements:
A: m is a positive integer, B: m^2 is divisible by 3
C: m is divisible by 3.
We are supposed to show $(A\wedge B)\Rightarrow C$. The "proof" begins with assuming C and ends with deducing C. It is proved that $C\Rightarrow C$ and not $(A\wedge B)\Rightarrow C$.

5) The proof is correct.
Denote statements:
A: π is irrational B: $\pi - x$ is irrational C: $\pi + x$ is irrational
We know A is true we want to show that $B\vee C$. The "proof" shows that $\neg(B\vee C)\Rightarrow \neg A$ by using the tautology $\neg(B\vee C)\Leftrightarrow \neg B \wedge \neg C$. Thus $\neg(B\vee C)\Rightarrow \neg A \wedge A$. Hence $B\vee C$.
Everything is correct.