

1) Let  $A$  be a  $4 \times 4$  matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 7 & 8 \end{bmatrix}$$

(a) Find  $\det A$ . (b) Find  $\text{rank } A$  using determinants.

(c) Consider the system:

$$(NH) \begin{cases} x_1 + 2x_2 = 1 \\ 3x_1 + 4x_2 = 1 \\ 5x_3 + 6x_4 = 2 \\ 7x_3 + 8x_4 = 0 \end{cases}$$

That is,  $A \cdot \mathbb{x} = \mathbb{b}$  for  $\mathbb{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ ,  $\mathbb{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ .

Without solving the system, does (NH) have solutions and how many?

(d) How about the corresponding homogeneous system:

$$(H) \quad A \cdot \mathbb{x} = \mathbf{0} \quad ?$$

2) # 12, p. 314

3) # 20 p. 314

4) # 10, p. 323