#2. Define:
\[ F_1 = A_1, \quad F_n = A_n \cup A_{n-1}, \quad \text{for } n=2,3,\ldots \]
As in class, we prove that the family \( \{ F_n \}_{n=1}^\infty \) is disjoint and \( \bigcup_{n=1}^\infty F_n = \bigcup_{n=1}^\infty A_n \).
From countable additivity of the measure, we obtain:
\[
m(\bigcup_{n=1}^\infty A_n) = m(\bigcup_{n=1}^\infty F_n) = \sum_{n=1}^\infty m(F_n) = \lim_{n \to \infty} \sum_{k=1}^n m(F_k). \quad (1)
\]
But
\[
\sum_{k=1}^\infty m(F_k) = m(\bigcup_{k=1}^\infty F_k) = m(\bigcup_{k=1}^\infty A_k) = m(\bigcup_{k=1}^\infty A_k). \quad (2)
\]
(The first equality follows from additivity of \( m \), the second from the definition of \( F_k \)'s, the last from the fact that \( A_k \subseteq A_{k+1}, \ k=1,2,\ldots \))

(1) and (2) imply
\[
m(\bigcup_{n=1}^\infty A_n) = \lim_{n \to \infty} m(\bigcup_{k=1}^n A_k).
\]

#1 Proof straightforward from #2. Indeed, let \( B_n = \bigcup_{i=1}^\infty A_i, \ n=1,2,\ldots \)
Then \( B_{n+1} \supseteq B_n, \ B_n \in \mathcal{M}, \ n=1,2,\ldots \) and \( \bigcup_{n=1}^\infty B_n = \bigcup_{n=1}^\infty A_n \).
Thus,
\[
m(\bigcup_{n=1}^\infty A_n) = \lim_{n \to \infty} m(\bigcup_{i=1}^n A_i).
\]

#3. Note that
\[
E_i \cup E_2 = (E_i \cap E_2) \cup (E_2 \setminus E_1) = (E_2 \setminus E_1), \quad E_i = (E_2 \setminus E_1), \quad E_2 = (E_2 \setminus E_1)
\]
and the collections on the right-hand side of each equality are disjoint. From additivity of \( m \), we obtain:
\[
m(E_i \cup E_2) = m(E_i \cap E_2) + m(E_2 \setminus E_1) + m(E_i \cap E_1).
\]
Thus:
\[
m(E_i \cup E_2) + m(E_2 \setminus E_1) = (m(E_i \cap E_2) + m(E_2 \setminus E_1)) + m(E_2 \setminus E_1) = m(E_i) + m(E_2)
\]

#4 Done in class.
As \( \bigcap_{i=1}^{\infty} E_i = \bigcup_{i=1}^{\infty} (A \cap E_i) \) from subadditivity of \( m^* \) we obtain

\[
m^* (\bigcap_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^* (A \cap E_i). \tag{3}
\]

From Lemma 11.2 we have for every \( k = 1, 2, \ldots \):

\[
m^* (\bigcap_{i=1}^{k} E_i) = \sum_{i=1}^{k} m^* (A \cap E_i).
\]

Since \( \bigcap_{i=1}^{\infty} E_i \supseteq \bigcap_{i=1}^{k} E_i \), we have:

\[
m^* (\bigcap_{i=1}^{\infty} E_i) \geq \sum_{i=1}^{k} m^* (A \cap E_i).
\]

Since the latter inequality holds for every \( k = 1, 2, \ldots \), we have

\[
m^* (\bigcap_{i=1}^{\infty} E_i) \geq \sum_{i=1}^{\infty} m^* (A \cap E_i). \tag{4}
\]

Thus, \( m^* (\bigcap_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} m^* (A \cap E_i) \) by (3) and (4).