#2) From monotonicity of the outer measure:

$$m^*(A \cup B) \geq m^*(B).$$

From subadditivity:

$$m^*(A \cup B) \leq m^*(A) + m^*(B) = m^*(B).$$

Thus, $m^*(A \cup B) = m^*(B)$.

#4) Let $\varepsilon > 0$ be fixed. From the definition of m^*, there exists a Lebesque covering $\{I_n\}$ of A such that

$$m^*(A) \leq \sum_n l(I_n) \leq m^*(A) + \varepsilon.$$

Let $\mathcal{U} = \bigcup_n I_n$. Then $\mathcal{U}$ is open as all I_n are open intervals. Also, $\{I_n\}$ is a Lebesgue covering of $\mathcal{U}$. Thus:

$$m^*(\mathcal{U}) \leq \sum_n l(I_n) \leq m^*(A) + \varepsilon.$$

As $A \subseteq \bigcup_n I_n$, $A \subseteq \mathcal{U}$.

#5) We use the previous problem. For every $n = 1, 2, \ldots$, there exists an open set $\mathcal{U}_n$ such that

$$A \subseteq \mathcal{U}_n, \qquad m^*(\mathcal{U}_n) \leq m^*(A) + \frac{1}{n} \quad (1)$$

Let $G = \bigcap_{n=1}^{\infty} \mathcal{U}_n$. Then G is a G_δ set as the intersection of a countable family of open sets. Since $G \subseteq \mathcal{U}_n$ for every $n = 1, 2, \ldots$, (1) gives:

$$A \subseteq G, \qquad m^*(G) \leq m^*(\mathcal{U}_n) \leq m^*(A) + \frac{1}{n} \text{ for } n = 1, 2, \ldots$$

Thus, $m^*(G) = m^*(A)$.